Effective action for a vector field in the theory of open superstrings

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A closed expression is derived for those terms in the effective action corresponding to the theory of open superstrings which depend on the (Abelian) strength of the vector field. The result is a modification of the Born-Infeld action, which arises in the case of open Bose strings.

Key problems in superstring theory, currently regarded as a strong candidate for a unified theory of all interactions,[1,2] are the problem of finding the ground state and that of finding a low-energy correspondence with the field-theory models. An important role in the solution of these problems is played by the effective action for fields corresponding to massless excitations of a superstring.[3–8] This action may be thought of as a one-parameter (α') generalization of the action of the $D = 10$ super-Yang-Mills theory and a $D = 10$ supergravity. So far, we know only the leading terms in the effective action; its general structure remains unknown. In the present letter we take a first step toward an understanding of this structure. Specifically, we derive a closed expression (in all orders in α') for terms which depend on the strength of the vector field in the effective action in the theory of open superstrings (of type 1). Although the theory of type 1 superstrings is apparently finite and free of anomalies in the case of the $G = SO(32)$ gauge group,[1,9] we find a closed expression for the effective action only in the Abelian limit (i.e., by choosing a background vector field which belongs to an Abelian subgroup of G). In the non-Abelian case, we find only terms of order up to α'^4 inclusively.

We denote by $I_0[x^\mu, g_{ab}, \theta^\alpha_A]$ the covariant action of the superstring[10] ($\mu = 0,...,9$; $a,b,A = 1,2$; $\alpha = 1,...,32$). At the boundary of the world surface we have $\theta_1 = \theta_2 = \theta$, where θ is the Majorana-Weyl spinor. We define the quantity[7]

$$\Gamma[\mathcal{A}] = g^{-2}\int dg_{ab}\, dx^\mu d\theta^\alpha_A \exp(iI_0)\mathrm{tr}(P\exp(-\int d\tau \dot{\Sigma}^M \mathcal{A}_M(\Sigma))) , \qquad (1)$$

where the integration is over surfaces with the topology of a disk (we are restricting the discussion to the tree approximation), g is a dimensionless coupling constant, P is the ordering along the boundary, $\Sigma^M = \{x^\mu, \theta\}$, and $\mathcal{A}_M$ is the superfield of the vector potential in the $D = 10$ super-Yang-Mills theory, which belongs to the algebra of the group G [the trace (1) is taken over the fundamental representation of G]. The superfield $\mathcal{A}_M$ satisfies the familiar constraints; i.e., is expressed in terms of the component fields A_μ and λ^α, which satisfy classical equations of motion.[7] In a case in which (1) is calculated through an expansion in powers of $\mathcal{A}$, and A_μ and λ^α are chosen to belong to the mass shell, Γ has the meaning of a generating functional for the scattering amplitudes of massless modes in the theory of open superstrings. If (1) is instead calculated through an expansion in derivatives of $\mathscr{A}$ (through an expansion in α'), i.e., if we distinguish a point y, $x^\mu = y^\mu + \xi^\mu$, $y^\mu = \text{const}$, around which we expand the fields, then the expression for Γ, $\Gamma = \int d^D y \mathscr{L}(A, \partial A, ...)$, is local, and Γ has the meaning of an effective action[1)] (Refs. 6–8). The integrand in (1) has a local fermion invariance which allows us to eliminate half of the components θ_A through the choice of a "light gauge"[10,7,11] $\gamma^+ \theta_A = 0$ along with $\sqrt{g} g^{ab} = \eta^{ab}$, $x^+ = y^+ + \tau$. Assuming $\lambda = 0$ and that the background vector field A_μ is nonzero only for $\mu = i = 1,...,8$, that it depends on only the "transverse" coordinates x^i, and that it corresponds to an Abelian subgroup of G, we find the following results for the total (Euclidean) action in (1) (the action of a superstructure which is interacting with an external vector field): $I = I_0 + I_{\text{int}}$, where

$$I_0 = \frac{1}{2} \int d^2 z \, (\partial_a \xi^i \partial_a \xi^i + Q^n \bar{\partial} Q^n + \bar{Q}^n \partial \bar{Q}^n),$$
$$I_{\text{int}} = i \int dt \, (\dot{\xi}^i \tilde{A}_i (y + \xi) - \frac{1}{8} S \gamma^{ij} S \tilde{F}_{ij} (y + \xi)). \tag{2}$$

Here we have switched to Euclidean notation ($\tau = -it$, $z = t + i\sigma$, $\partial = \partial / \partial z$); $\tilde{A}_i = 2\pi\alpha' A_i$; $\tilde{F}_{ij} = \partial_i \tilde{A}_j - \partial_j \tilde{A}_i$; and Q^n is a complex fermion variable which parametrizes the independent components of θ and is an SO(8) spinner ($n = 1,...,8$). At the boundary we have $\text{Re} Q^n = S^n$. The quantity I_{int} (2) corresponds to a vertex operator from Ref. 12. Expanding A around y, and integrating over ξ and Q, we find in the general case a Γ which depends on the strength F and all its derivatives. Restricting the analysis to a calculation of the F dependence of the effective action, we can omit from (2) all terms containing derivatives of[2)] F. In this case we have

$$I_{\text{int}} = \frac{i}{2} \tilde{F}_{ij} \int dt \, \xi^i \dot{\xi}^j - \frac{i}{2} \hat{F}_{nm} \int dt S^n S^m, \qquad \hat{F}_{nm} = \frac{1}{4} \gamma^{ij}_{nm} \tilde{F}_{ij}. \tag{3}$$

The expectation values $\langle V \rangle$ of the "vertex operators" (3) are identically zero, so that the dependence on the conformal factor of the metric (1) drops out entirely at $D = 10$ (the result for Γ does not depend on the choice of a "Weyl gauge"). The remaining integral over ξ and Q is Gaussian and can be evaluated exactly. [The first step is to reduce the integral over ξ and Q to an integral over their values at the boundary of a unit disk:

$$\int d\xi dS \exp(-(1/2)\xi \Delta_B \xi - (1/2) S \Delta_F S),$$

where

$$\Delta_B = (1/\pi) \sum_{n=1}^{\infty} n \cos n (t - t')$$

and $\Delta_B = \dot{\Delta}_F$. We then "diagonalize" $\tilde{F}_{ij}$ and $\hat{F}_{nm}$, putting them in block form. Finally, we use $\Pi_{k=1}^{\infty} c = c^{-1/2}$.] As a result ($D = 10$), we find

$$\Gamma(F) = N g^{-2} (2\pi\alpha')^{-D/2} \int d^D y \, \mathcal{L}_S(F), \qquad \mathcal{L}_S = [\det(\delta_{ij} + \tilde{F}_{ij}) / \det(\delta_{nm} + \hat{F}_{nm})]^{1/2}, \tag{4}$$

where $N = \mathrm{tr}1 = 32$. The numerator in $\mathscr{L}_S$ is the contribution of bosons, while the denominator is that of fermions. In the case of boson strings, the calculation can be carried out in an explicitly $O(10)$-invariant form, with a result in the form of a Born-Infeld Lagrangian:

$$\mathcal{L}_B = [\det(\delta_{\mu\nu} + \widetilde{F}_{\mu\nu})]^{1/2} = 1 + \frac{1}{4}J_1 - \frac{1}{8}J_2 + \ldots, \tag{5}$$

$$J_1 = \widetilde{F}^2_{\mu\nu}, \quad J_2 = (\widetilde{F}_{\mu\lambda}\widetilde{F}_{\nu\lambda})^2 - \frac{1}{4}(\widetilde{F}_{\mu\nu}\widetilde{F}_{\mu\nu})^2, \quad \widetilde{F}_{\mu\nu} = 2\pi\alpha' F_{\mu\nu}. \tag{6}$$

Expression (4) allows the obvious $O(10)$-invariant generalization, since it does not contain an 8-index ϵ-tensor. In the $\mathscr{L}_S = 1 - (3/16)J_2 + \ldots$, the $F^2_{\mu\nu}$ terms, which arise from bosons and fermions, cancel out. Incorporating the $SL(2,R)$-invariance on the disk shows that this calculation cannot reveal the coefficient of the $F^2_{\mu\nu}$ term, which must be determined through a separate calculation in the non-Abelian case (more on this below).

Let us compare (4) and (5) with the effective action which can be reconstructed from the known[12] 3- and 4-particle tree scattering amplitudes on the mass shell. We find that in the case of a boson string the corresponding non-Abelian effective Lagrangian is[3)]

$$\begin{aligned}\mathcal{L}_B = {} & a_0 \mathrm{tr}\,\{F^2_{\mu\nu} + 2\alpha'[a_1 F_{\mu\nu}F_{\nu\rho}F_{\rho\mu} + a_2(D_\lambda F_{\lambda\mu})^2] \\ & + (2\alpha')^2[a_3 F_{\mu\lambda}F_{\nu\lambda}F_{\mu\rho}F_{\nu\rho} + a_4 F_{\mu\lambda}F_{\nu\lambda}F_{\nu\rho}F_{\mu\rho} + a_5(F_{\mu\nu}F_{\mu\nu})^2 + a_6 F_{\mu\nu}F_{\lambda\rho}F_{\mu\nu}F_{\lambda\rho} \\ & + a_7 F_{\mu\nu}F_{\nu\rho}D^2F_{\rho\mu} + a_8 F_{\mu\nu}D_\lambda F_{\mu\nu}D_\rho F_{\rho\lambda} + \\ & + a_9 D_\lambda F_{\lambda\mu}D_\rho F_{\rho\nu}F_{\mu\nu} + a_{10}D_\rho D_\lambda F_{\lambda\mu}D_\rho D_\sigma F_{\sigma\mu} + a_{11}(D^2F_{\mu\nu})^2] + O(\alpha'^3)\},\end{aligned} \tag{7}$$

where

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + [A_\mu, A_\nu], \; D_\mu = \partial_\mu + [A_\mu,],$$

$$a_0 = -(1/8)(2\pi\alpha')^2, \; \alpha_1 = (4/3), \; a_2 = -2, \; a_3 = (\pi^2/3) - \beta,$$

$$a_4 = (\pi^2/6) + \beta, \; a_5 = -(\pi^2/12) - (1/2), \; a_6 = -(\pi^2/24) + (1/2),$$

$$a_7 = (1/2)\beta - 4a_{11}, \; a_8 = (1/2)\beta + 4a_{11}, \; a_{10} = -2a_{11} - (1/2).$$

To determine the constants β (or a_8), a_9, and a_{11}, we need to examine the 5-particle amplitude. The derivative-independent terms in the Abelian limit of (7) agree with (5). In the case of superstring theory, we find (7) with

$$a_1 = a_2 = 0, \; a_3 = (\pi^2/3) - \beta, \; a_4 = (\pi^2/6) + \beta, \; a_5 = -(\pi^2/12),$$

$$a_6 = -(\pi^2/24), \; a_7 = (1/2)\beta - 4a_{11}, \; a_8 = (1/2)\beta + 4a_{11}, \; a_{10} = -2a_{11}.$$

To reconcile the result with (4), we must add to (4) an $\widetilde{F}^2_{\mu\nu}$ term with a coefficient of 3/8. The kinetic operator in (7) is $K = (-\Box)P$, where

$$P = 1 - \alpha' a_2 \Box + 2\alpha'^2(a_{10} + 2a_{11})\Box^2 + O(\alpha'^3),$$

so that the "propagator corrections" are absent ($P=1$) only in the case of superstring theory. In the theory of a boson string, the function $P(-\Box)$ (which is calculated exactly in terms of α') should apparently not have any zeros on the complex plane.

We can also generalize (4) and (5) to the single-loop level. In the boson case, the quadratic divergence of the integral over the modular parameter is absorbed in the renormalization of the constant $1/g^2$ in front of the Born-Infeld tree term. In the superstring case, the logarithmic divergences of the contributions of the Möbius sheet and ring cancel each other out for $G = SO(N)$, $N = 32$ (by analogy with the cancellation of divergences in the amplitudes.[1,9]).

[1)]In carrying out an expansion around a single point (and renormalizing with the help of the counterterms of the two-dimensional theory), we are automatically taking a "Legendre transformation": We are omitting nonlocal contributions of the diagrams which stem from exchanges of massless particles.

[2)]In other words, we can assume that we are dealing with a background with a constant strength $F=\text{const}$, which is the solution of the classical Yang-Mills equations (in the Abelian case). The satisfaction of these equations is necessary for the consistency of (1).

[3)]The presence of $F^2$and F^3 terms (7) has been noted previously.[3,4] The appearance of $(DF)^2$ term stems from the existence of a tachyon in the theory of a Bose string and indicates that the assertion[13] that the effective action should not contain "propagator" terms is erroneous. That assertion is valid only in a superstring theory (e.g., in the effective action of the theory of closed Bose strings, the R^2 terms do not form a "Gauss-Bonn" combination".

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