Quantum strings with a dynamic geometry

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A new theory of boson strings is proposed. The geometry of the surface of a string of the type under consideration here is described by a metric and a twisting. The corresponding effective Lagrangian depends on scalar and vector fields. The question of the quantization of this string in a space of arbitrary dimensionality is examined. A condition under which the energy is positive definite is derived.

Superstring theory is currently regarded as the basis of a unified theory of elementary particles. String theory also plays an important role in statistical physics.[1–6] The boson part of all these theories is identical and has an action proportional to the surface area of the string. Alternatively, at the classical level, the equivalent Lagrangian is[7,8]

$$L_0 = \frac{1}{2}\sqrt{-g}\, g^{\alpha\beta}\partial_\alpha X^\mu \partial_\beta X^\mu, \quad g = \det g_{\alpha\beta},$$

where $X^\mu(\zeta)$, $\mu = 0,1,...,D-1$, are the coordinates of the string, and ζ^α, $\alpha = 0,1$ are parameters on the string. In this case the metric does not have a dynamics on the string, $g_{\alpha\beta}(\zeta)$.

In the present letter we propose a new theory of boson strings. In this theory, the surface of the string has a twisting, and the geometry becomes dynamic even at the classical level. In other words, both the metric and the twisting obey second-order equations of motion. We know that the addition of a scalar curvature R to L_0 does not lead to dynamic equations for the metric $g_{\alpha\beta}$, since in the two-dimensional case $\sqrt{-g}R$ has the form of a total divergence. We will show that the effective Lagrangian of a quantum string with a dynamic geometry, in contrast with a theory with the Lagrangian L_0, allows a simple vacuum solution, so that there is the hope that the theory for such a string will be free of the difficulties which stem from the appearance of a tachyon and a critical dimensionality $D = 26$.

We describe the surface of the string by means of the field of reference marks e_α^a, $a = 0,1$, $g_{\alpha B} = e_\alpha^a e_{\beta a}$, and the Lorentzian connection $\omega_\alpha^{ab} = -\omega_\alpha^{ba}$. The curvature and twisting tensors are

$$R_{\alpha\beta}{}^{ab} = \partial_\alpha \omega_\beta{}^{ab} - \omega_\alpha{}^{ac}\omega_{\beta c}{}^{b} - (\alpha \leftrightarrow \beta), \quad T_{\alpha\beta}{}^{a} = \partial_\alpha e_\beta{}^{a} - \omega_\alpha{}^{ab} e_{\beta b} - (\alpha \leftrightarrow \beta),$$
$$R^{abcd} = R_{\alpha\beta}{}^{cd} e^{\alpha a} e^{\beta b}, \qquad T^{abc} = T_{\alpha\beta}{}^{c} e^{\alpha a} e^{\beta b}, \qquad R = R_{ab}{}^{ab}.$$

We consider the most general P-even Lagrangian, quadratic in the curvature and twisting tensors, which describes the dynamics of the internal geometry of the string:

$$L_1 = \sqrt{-g}\left(\frac{1}{4}\mu^2 R^2_{abcd} + \frac{1}{4}\gamma^2 T^2_{abc} + \lambda\right). \tag{1}$$

Of the 10-parameter family of Lagrangians in 4-dimensional space,[9] we are left in the 2-dimensional case with only a 3-parameter family with the constants μ, γ, and λ.

The total action for the string is

$$S = \int d^2\zeta L, \qquad L = L_0 + L_1 . \tag{2}$$

In the conformal gauge $e^a_\alpha = e^\varphi \delta^a_\alpha$, the parametrization $\omega^{ab}_\alpha = A_\alpha \epsilon^{ab}$ $\cdot(\epsilon^{ab} = -\epsilon^{ba})$ reduces Lagrangian (1) to the expression

$$L_1 = -\frac{\mu^2}{2} e^{-2\varphi} F^2_{\alpha\beta} + \frac{\gamma^2}{2}[(\partial_\alpha\varphi)^2 + F_{\alpha\beta}\epsilon^{\alpha\beta}\varphi - A^2_\alpha] + \lambda e^{2\varphi}, \tag{3}$$

where $F_{\alpha\beta} = \partial_\alpha A_\beta - \partial_\beta A_\alpha$. This Lagrangian describes the interaction of a scalar field φ with the massive vector field A_α. If we introduce the extension $\varphi \to \gamma\varphi$, $A_\alpha \to \mu A_\alpha$ in Lagrangian (3), and if we take the limit $\gamma/\mu \to 0$, we find

$$L_1 = -\frac{1}{2} e^{-2\varphi/\gamma} F^2_{\alpha\beta} + \frac{1}{2}(\partial_\alpha\varphi)^2 + \lambda e^{2\varphi/\gamma}. \tag{4}$$

It can be shown that Lagrangian (4) specifies a completely integrable system which reduces to a Liouville equation. In contrast with the Liouville equation, however, there is a simple vacuum solution for Lagrangian (4) in the Euclidean signature:

$$\varphi = \varphi_0 = \text{const}, \qquad F_{\alpha\beta} = \sqrt{|\lambda|}\epsilon_{\alpha\beta} e^{2\varphi_0/\gamma} . \tag{5}$$

A quantization of the string theory with action (2) on the basis of a functional integration over the surfaces[10] leads to an effective Lagrangian of the type in (3), (4). A Lagrangian of the type in (4) can be quantized near vacuum (5) in any dimensionality D.

For a canonical quantization of the string theory we need to construct a Hamiltonian formalism. As a first step in this direction, we examine the Lagrangian L_1, which is also of interest as a model of a two-dimensional gravity with a dynamic twisting. Other models of two-dimensional gravity have been exmained in Refs. 11 and 12, among other places.

In the time gauge $e^a_\alpha = \text{diag}\,(1,e)$, with the constraints, the canonical Hamiltonian reduces to the expression

$$H = \int d\zeta^1 \mathcal{H}, \qquad \mathcal{H} = \frac{e}{4\mu^2}(\pi^1)^2 + \frac{e}{2\gamma^2}\pi^2 - \pi A_1 - \frac{1}{2\gamma^2 e}(\partial_1\pi^1)^2 - \lambda e,$$

where π and π^α are generalized momenta which are the conjugates of e and A_α, respectively. Here $\pi^0 = 0$. The Hamiltonian density $\mathcal{H}$ is not positive definite. The equations of motion for the component e_{00} of the tetrad field are

$$\mathcal{H} - \gamma^2\partial_1 A_0 = 0.$$

The total energy on the equations of motion,

$$E = \int_a^b d\zeta^1 \mathcal{H} = -\frac{\mu^2}{e} \partial_1 R \Big|_a^b \,, \tag{6}$$

is therefore determined by the boundary behavior of the dynamic variables. In particular, for two-dimensional models of a closed universe, to which closed strings correspond, and also for an open universe with a sufficiently rapid decay of the derivative of the curvature at infinity, we have $E=0$. It would be of interest to study the two-dimensional analog of the plane spaces with $E>0$.

The total energy for action (2), with allowance for the coordinates of the string, is of the same form as (6).

In summary, we have proposed a string theory in which both the metric and the twisting are dynamic variables. There is the hope that such strings will make it possible to carry out a quantization in a space of arbitrary dimensionality.

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