Wave precursors of quasiparallel shock waves

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Cyclotron absorption of wave energy by resonant ions at the leading edge of a shock wave gives rise to a precursor in the form of an intense magnetosonic wave.

Since the dispersion of waves forming a quasiparallel wave is positive, an oscillatory structure forms in the bow part of the wavefront.[1] The region in which this structure exists is usually limited in size to a few ion Larmor radii, i.e., distances over which dissipative processes stemming from either a firehose instability[2] or the presence of beams reflected from an ion wavefront[3] are important.

In the present paper we consider the cases in which high-energy ions formed in the bow part of the front have a stable velocity distribution function. We show that the additional dissipation, which results from the cyclotron absorption of wave energy by these ions, gives rise to an outgoing precursor of the shock wave which propagates far out in front. This precursor is in the form of rather intense magnetosonic waves. The structure of the shock wave in this case is shown in Fig. 1.

We begin our analysis with the following equation for the perturbation of the magnetic field in a quasiparallel shock wave:

$$\frac{c^2}{\omega_{pi}^2}\frac{d^2H_x}{d\zeta^2} + \left(1 - \frac{v_A^2}{v_{\rm ph}^2}\right)H_x + \frac{\theta}{H_0}H_x^2 + \frac{\nu c^2}{\omega_{pe}^2}\frac{1}{v_{\rm ph}}\frac{dH_x}{d\zeta} + \frac{4\pi}{c}\int_{\infty}^{\zeta} J_y^{res}\, d\zeta' = 0. \qquad (1)$$

Here $\zeta = \mathbf{e}(\mathbf{r} - \mathbf{v}_{\rm ph} t)$; $\mathbf{e}$ is the unit vector along the wave propagation direction in the xz plane; and θ is the angle between $\mathbf{e}$ and the z axis, which runs along the unperturbed magnetic field H_0. For quasiparallel shock waves we would have $\theta \ll 1$, where $v_A = H_0/\sqrt{4\pi n_0 m_i}$ is the Alfvén velocity. It is assumed here that the characteristic frequency (ω) of the wave process in (1) is substantially lower than the ion cyclotron frequency ω_{Hi}, here ω_{pe} and ω_{pi} are the electron and ion plasma frequencies, respec-

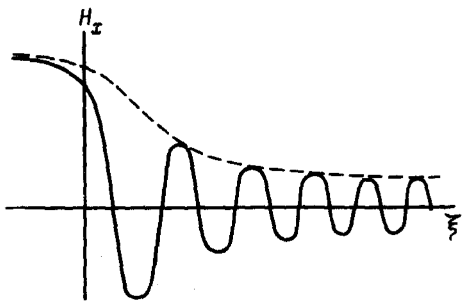

FIG. 1.

tively. The last two terms in Eq. (1) describe a dissipation which results from either effective collisions (ν is the collision rate) or resonant particles ($J^{\rm res}$ is the flux of resonant particles). If these terms are ignored, Eq. (1) has the integral

$$G = \frac{c^2}{2\omega_{pi}^2}\left(\frac{dH_x}{d\zeta}\right)^2 + V(H_x) = \text{const}, \tag{2}$$

which corresponds to conservation of the energy of a nonlinear oscillator which is oscillating in a potential well

$$V(H_x) = \frac{\delta}{2}H_x^2 + \frac{\theta}{3H_0}H_x^3, \qquad \delta = 1 - \frac{v_A^2}{v_{\rm ph}^2}.$$

For a shock wave, the Mach number is thus $M = v_{\rm ph}/v_A > 1$, and we have $\delta > 0$. The potential well $V(H_x)$ in Fig. 2 corresponds to the case $\theta < 0$; the maximum potential energy is reached at $H_m = -(\delta/\theta)H_0$. In the case $\theta > 0$, then solution is found through the substitution $H_x \to -H_x$. The dissipation of wave energy gives rise to a slow change [slow in comparison with the period of $H_x(\zeta)$] in G in accordance with the equation

$$\frac{dG}{d\zeta} = -\frac{\nu c^2}{\omega_{pe}^2}\left\langle\left(\frac{dH_x}{d\zeta}\right)^2\right\rangle - \frac{4\pi}{c}\left\langle\frac{dH_x}{d\zeta}\int_{\infty}^{\zeta} j_y^{res}\, d\zeta'\right\rangle. \tag{3}$$

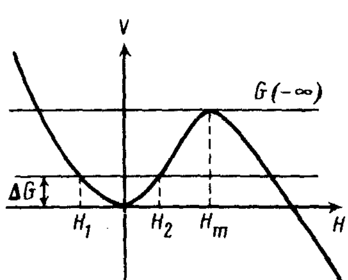

FIG. 2.

The angle brackets on the right side of this equation mean an average over the period of $H_x(\zeta)$. The shock-wave structure in Fig. 1 corresponds to the condition $G(\zeta\to\infty)\to 0$. Because of dissipation, the value of G increases with decreasing ζ, to the value $G(\zeta\to-\infty)=V(H_m)$, with the consequence that the magnetic field oscillates with increasing amplitude. It asymptotically reaches the value $H_x=H_m$ in the limit $\zeta\to-\infty$. At small values $G(G\ll\delta H_m^2)$, the solution in the precursor region can be written in the form

$$H_x(z,t)=\sqrt{\frac{2G}{\delta}}\sin(k_0z-\omega t),\qquad k_0=\frac{\omega_{pi}}{c}\sqrt{\delta},\quad \omega=k_0v_A. \tag{4}$$

Since only the dissipation due to resonant particles is important in the precursor region, the equation for G leads to the following equation (Ref. 4, for example):

$$\frac{dG}{d\zeta}=\sqrt{\frac{2G}{\delta}}\,\frac{e\omega}{2c}\int d\mathbf{v}_0dt_0f_0(\mathbf{v}_0)[\cos(k_0\zeta-\varphi(\zeta,t_0,\mathbf{v}_0))-\cos(k_0\zeta+\varphi(\zeta,t_0,\mathbf{v}_0))]. \tag{5}$$

Here t_0 is the time at which a resonant particle arriving at the time t at the point ζ, v in phase space enters the interaction region; $\mathbf{v}_0$ is the initial velocity of the particle; $f_0(\mathbf{v}_0)$ is the distribution function of the entering particles; and φ is the azimuthal angle in the v_x, v_y plane. To find the trajectories of the resonant particles, $\varphi(\zeta,t_0,v_0)$, we need to integrate the equation of motion. These trajectories turn out to be particularly simple in the case of a small oscillation amplitude of the magnetic field,[4]

$$\frac{e}{m_ic}\sqrt{\frac{2G}{\delta}}\ll\frac{k^3v_\perp^3}{\omega_{Hi}^2}$$

$$\frac{dv_z}{dt}=-\frac{e}{2m_ic}\sqrt{\frac{2G}{\delta}}v_\perp[\cos(k_0\zeta-\varphi)-\cos(k_0\zeta+\varphi)],\ v_\perp=\text{const},\ \frac{d\varphi}{dt}=-\omega_{Hi} \tag{6}$$

In the numerical integration of Eqs. (5) and (6), we consider the case in which the resonant particles are moving toward the wavefront ($v_z<0$), so that only the resonance corresponding to the normal Doppler effect, $\omega-kv_z\simeq\omega_{Hi}$, is important, and the initial distribution function of the resonant particles, $f_0(\mathbf{v}_0)$ is Maxwellian. The generalization to the case of an arbitrary distribution function is obvious. We use the dimensionless variables

$$\xi=\kappa\zeta,\ q=\frac{d\tau}{d\xi},\quad \nu=\frac{v_\perp}{\sqrt{T/m}},\quad h=\sqrt{\frac{G}{2\delta}}\,\frac{1}{H_0}\,\frac{k_0^3\sqrt{T/m}}{\omega_{Hi}\kappa^2}. \tag{7}$$

Here $\tau=k_0\zeta-\varphi$ is the phase of the wave field acting on the particle. The resonant velocity v_z can then be written in the form

$$v_z=\frac{\omega-\omega_{Hi}}{k_0}-\frac{\omega_{Hi}}{k_0^2}\frac{d\tau}{d\zeta},$$

where

$$\kappa = \sqrt{\frac{\pi}{128}}\, \frac{\omega_{Hi}}{v_A}\, \frac{1}{\sqrt{\beta \delta^3}}\, e^{-(1/2\delta\beta)} \approx \frac{\gamma^+}{v_A \delta}$$

is the spatial gain of G for a Maxwellian distribution of resonant particles, γ^+ is the damping constant in this case, and $\beta = 4\pi n_0 T / H_0^2$. In choosing units for G, we made use of the circumstance that the maximum magnetic-field amplitudes of the wave, at which cyclotron absorption does not occur, are determined from the known condition[4] $\Omega_{TR}/v_z \simeq \kappa$. Here

$$\Omega_{TR} = \left| \frac{e}{m_i} k_0 \frac{v_\perp}{c} \sqrt{\frac{2G}{\delta}} \right|$$

is the frequency of the phase oscillations of the resonant particles. Using an estimate of the spatial gain of G in (5), $\kappa \simeq (\omega_{Hi}/v_A \delta)(n'/n_0)$ (n' is the density of resonant ions), we easily find the following approximate expression for the magnetic field amplitude in the precursor:

$$\frac{H'_m}{H} \approx \left(\frac{n'}{n_0}\right)^2 \frac{1}{\delta^3}\ .$$

In terms of the dimensionless variables in (7), system (5), (6) can be rewritten in the following form after the replacement $\xi \rightarrow -\xi$:

$$\frac{d^2\tau}{d\xi^2} = h\nu \cos\tau; \qquad \frac{dh}{d\xi} = \frac{1}{\pi^2} \int d\tau_0 dq_0 d\nu_0 q_0 \nu_0 e^{-(\nu_0^2/2)} \cos\tau(\xi, \nu_0, q_0, \tau_0)\,. \qquad (8)$$

This system of equations has been integrated numerically for $\xi > 0$ for an ensemble of 10^4 resonant particles with the coordinates

$$0 < \tau_0 < 2\pi,\ -12 < q_0 < 12,\ -1 < \nu_0 < 1$$

and the initial condition $h(\xi = 0) = 10^{-2}$. The functional dependence $h(\xi)$ found as a result of the integration is shown in Fig. 3.

In interpreting observational data one should bear in mind that there is an alter-

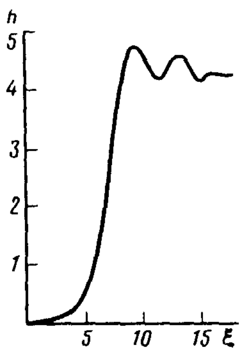

FIG. 3.

native mechanism for the excitation of Alfvén waves ahead of a shock front. This mechanism involves an instability which develops as a result of the appearance of a beam of reflected ions in the solar wind.[5,6] In this case, the Alfvén waves are irregular in nature and are worn down by the solar wind. The characteristic energy of the resonant ions that excite these waves is $\mathscr{E}^{\rm res} \simeq [(\omega^2_{Hi})/(2\omega^2)] m_i v_{sw}^2$ (ω is the frequency of the Alfvén waves, measured by the space vehicle), i.e., higher by a factor $(v_{sw}/v_A)^2$ than in the case discussed here, in which the wave precursor is tied to the shock front.

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