Excitation of a short-wavelength spectrum of surface waves in a plasma by an intense electromagnetic radiation

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(Submitted 16 January 1986)
Pis'ma Zh. Eksp. Teor. Fiz. **43**, No. 5, 237–239 (10 March 1986)

The modulation instability of the surface charge density oscillations, which develops as a result of application of an intense electromagnetic pump wave ($E_0^2 \gg 4\pi n_e T_e$) on the plasma, causes intense rf oscillations of the reflection factor of this wave. The energy of the small-scale excitations per unit area is on the order of $cE_0^2[m_i/m_e]^{1/3}/4\pi\omega$.

1. Strong parametric instabilities with high growth rates can develop when a strong electromagnetic pump field is applied to a plasma under the conditions in which the oscillation velocity of the electrons is higher than their thermal velocity.[1] These instabilities lead to a considerable deformation of the initial velocity distribution of the electrons and to an excitation of a broad spectrum of short-wavelength plasma oscillations[2,3] (see also Ref. 4). The application of an electromagnetic pump field on the plasma surface also causes a strong parametric instability with the excitation of short-wavelength surface plasma oscillations.[5] Aliev *et al.*[6] attributed the saturation of the parametric instability of this sort to the inverse effect of the excited short-wavelength spectrum on the pump wave which causes the total long-wavelength plasma-oscillation field to decrease. To estimate the saturation level, they assumed that the phase distribution in the short-wavelength spectrum is of a diffuse nature.

In this letter we will analyze the time-dependent dynamics of saturation of a strong parametric instability in which the surface waves are excited when an electromagnetic wave is incident along the normal to the plasma half-space. We will reject in this case the assumption that the phase distribution of the excited waves is random.

2. We assume that an electromagnetic wave with the field component $(0,H_z,E_z)$, where $|H_y| = |E_z| = E_0$, is incident along the normal to the plasma half-space ($x<0$) with an unperturbed constant plasma density n_e. We will also assume that the intensity of the field of the incident wave is reasonably large. We will ignore below the thermal spread of plasma electrons ($E_0^2 \gg 4\pi n_e T_e$, where T_e is the temperature of plasma electrons). We write the following system of equations[5] for the perturbed surface charge density $\sigma_\alpha = \lim_{\delta\to 0}\int_{-\delta}^{+\delta} n'_\alpha dx$ (e_α, m_α, and n'_α are the charge, the mass, and the perturbed charge density of the particles of α species):

$$\exp[ia_{\alpha n}\sin(\omega_0 t+\varphi)]\frac{\partial^2}{\partial t^2}\nu_{\alpha n} + \frac{\Omega_\alpha^2}{2}\sum_\beta \nu_{\beta n} = 0, \quad (1)$$

where

$$v_{\alpha n} = e_\alpha \sigma_{\alpha n} \exp[-a_{\alpha n} i \sin(\omega_0 t + \varphi)], \quad a_{\alpha n} = \frac{e_\alpha n E_z(k_z = 0)}{m_\alpha \omega_0 c}, \quad \Omega_\alpha^2 = \frac{4\pi e_\alpha^2 n_\alpha}{m_\alpha},$$

$(\omega_0 t + \varphi)$ is the phase of the field with $k_z = 0$ in the plasma, and ω_0 is the frequency of the incident wave. In accordance with Refs. 1 and 4, we seek the solution of (1) in the form of a series:

$$v_{\alpha n} = \sum_{s=-\infty}^{\infty} u_{\alpha n}^{(s)} \exp(is\omega_0 t).$$

We found that for our purpose we need to retain only the first term of the series for the surface perturbation of the ion density. The terms in v_{en}, which are proportional to $\exp(\pm i\omega_0 t)$, are considerably larger than the other terms of the series,[2-4] but the terms corresponding to the "zeroth" and second harmonics must also be retained. Let us consider the symmetric[1)] perturbations $u_{i,n}^{(0)} = u_{i,-n}^{(0)}$ (the wavenumber of the excited surface waves is $k_{zn} = n\omega_0/c$). In this case we have

$$u_{en}^{(0)} = u_{e,-n}^{(0)}, \quad u_{e,n}^{(\pm 2)} = u_{e,-n}^{(\pm 2)}, \quad u_{e,n}^{(\pm 1)} = -u_{e,-n}^{(\pm 1)}, \text{ and } [u_{e,n}^{(1)}]^* = u_{e,n}^{(-1)}.$$

The system of equations which takes into account the inverse effect of the field of the excited short-wavelength spectrum of the surface waves on the reflected wave (the parameters of the incident electromagnetic wave remain the same) is

$$\frac{du_{en}}{dt} + (\theta_n - i\Delta_1 \omega_0) u_{en} = i\frac{\omega_0}{2} J_1(a_n) u_{in} e^{i\varphi},$$

$$\frac{d^2 u_{in}}{dt^2} = -\omega_0 \frac{m_e}{m_i} J_1(a_n)[u_{en} e^{-i\varphi} + u_{en}^* e^{i\varphi}], \qquad (2)$$

$$D(R - R_0) = \frac{8\pi}{e n_e E_0} \sum_n u_{in} [J_0(a_n) u_{en}^* e^{i\varphi} - J_2(a_n) u_{en} e^{-i\varphi}],$$

where

$$1 + R = |1 + R| \exp(-i\varphi) = \frac{a_n}{\beta_0 n} \exp(-i\varphi), \quad \omega_0 = \frac{\Omega_e}{\sqrt{2}}(1 - \Delta_1), \quad \beta_0 = \frac{2eE_0}{m_e c \omega_0},$$

$$n = \frac{k_z c}{\omega_0}, \; R_0 = -\frac{D_0^*}{D_0}, \; D_0 = \frac{\epsilon_0}{\kappa_0} + i\frac{c}{\omega_0},$$

$$\epsilon_0 = 1 - \frac{\Omega_e^2}{\omega_0^2}, \; \kappa_0^2 = -\frac{\omega_0^2}{c^2}\epsilon_0, \; \Delta_1 = (m_e/m_i)^{1/3}\Delta.$$

Here R is the peak reflection factor, $u_{en}^{(1)} \equiv u_{en}$, and $u_{in}^{(0)} \equiv u_{in}$. The terms on the right side of the third equation in (2), which are proportional to $J_0(a_n)$ and $J_2(a_n)$, correspond to the contribution of the zeroth and second field harmonics, respectively, to the nonlinear interaction. From the system of equations in (2) we can derive the relation

$$1 - |R|^2 = \frac{16\pi}{e\beta_0 n_e c E_0} \sum_n \frac{1}{n}\left(\frac{d|u_{en}|^2}{dt} + 2\theta_n |u_{en}|^2\right), \qquad (3)$$

which is the energy-conservation law.

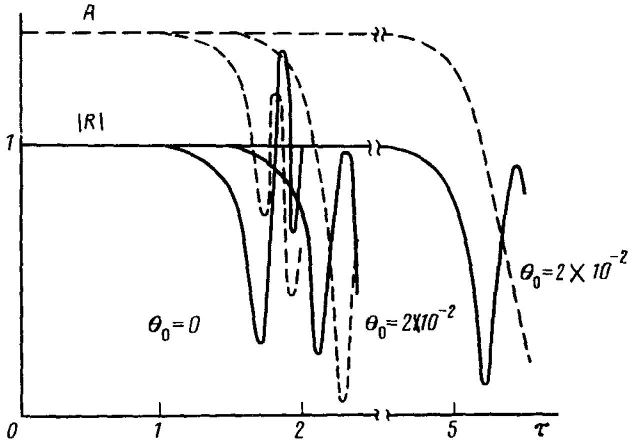

FIG. 1.

3. The system of equations (2) was solved numerically. Figure 1 shows the behavior of the modulus of the reflection factor $|R|$ as a function of time $\tau = (m_e/m_i)^{1/3}$ $\omega_0 t$ for different dissipation levels:

$$\theta_n = n\theta_0(m_e/m_i)^{1/3}\omega_0 \ \ (\theta_0 = 0,\ 2\times10^{-2}, 2\times10^{-3}, \Delta = 1/2,\ \beta_0 = 0.04,\ n_{\min} = 10,\ n_{\max} = 100).$$

According to Fig. 1, the mean intensity of the electric field, AE_0, with $k_z = 0$ decreases in the plasma. The instability saturation level estimated from the decrease in the integral field with $k_z = 0$ in the plasma[6] is thus in qualitative agreement with the results of the calculations given above. We should note, however, that the spread in the phases φ_n $[u_{en} = |u_{en}|\times\exp(i\varphi_n)]$ of the short-wavelength spectrum during the development of the instability decreases rapidly and is essentially absent in the nonlinear stage. Specifically, the nonlinear phase mode locking is the cause of the strong interaction of

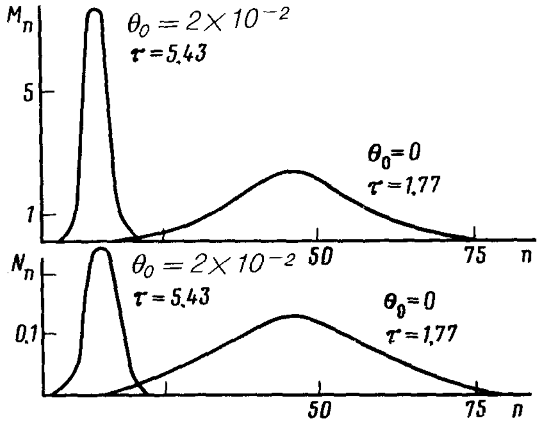

FIG. 2.

the pump wave and the short-wavelength spectrum. The rf oscillations of the modulus of the reflection factor $|R|$ can be seen in this case. On the other hand, phase locking of the spectrum gives rise to a small-scale spatial modulation of the surface charge density with a scale dimension on the order of $2\pi c/n_m \omega_0$, where n_m is the number of the mode with a maximum amplitude. The spectra of excitable surface waves are shown in Fig. 2 for the cases $\theta_0 = 0$ and $\theta_0 = 2\times 10^{-2}$ in the presence of a well-developed instability. The notation used in Fig. 2 is as follows:

$$N_n = \frac{4\pi}{E_0}\left(\frac{m_e}{m_i}\right)^{1/6} |u_{en}|\beta_0^{1/2}, \quad M_n = \frac{4\pi}{E_0} u_{in}\left(\frac{m_e}{m_i}\right)^{-1/6} \beta_0^{1/2}.$$

The counting was controlled under the energy conservation law in accordance with the procedure used in Ref. 3. The dissipation in the system considerably contracts the spectrum of the excitable waves, slightly lowers the integral level of their energy, and protracts the development of the instability. The scale dimension of the modulation of the surface charge density also increases. The total energy of the short-wavelength spectrum per unit surface area of the plasma $(\theta_0 = 0)$

$$\frac{cE_0^2}{4\pi}\int_0^t (1 - |R|^2)dt' \tag{4}$$

is on the order of $cE_0^2 (m_i/m_e)^{1/3}/4\pi\omega_0$. We note in conclusion that since the instability lasts only a very short time $[\sim\omega_0^{-1} (m_i/m_e)^{1/3}]$, the motion of the plasma boundary due to the rf pressure of the pump field can be ignored.

[1)] The same result can also be obtained, to within a certain change in notation, for asymmetric perturbations $u_{in}^{(0)} = -u_{i-n}^{(0)}$.

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Translated by S. J. Amoretty