Excitation and emission of a "detector" in accelerated motion in a vacuum or in uniform motion at a velocity above the velocity of light in a medium

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The question of the excitation of a "detector" moving at constant acceleration in a vacuum has recently been discussed extensively. In the present letter it is noted that this excitation and the associated emission are analogous to corresponding events in the region of the anomalous Doppler effect which arises when a "detector" moves at a constant velocity above the velocity of light in a medium.

The excitation of a "detector" (an oscillator, an atom, etc.) which occurs as the detector moves through vacuum at a constant acceleration a has been discussed extensively in the literature for a decade now, since the paper by Unruh.[1] In the steady state, the probability distribution of the detector over energy levels is the same as in the field of equilibrium thermal radiation with a temperature (k is the Boltzmann constant)

$$T = \hbar a / 2\pi k c . \qquad (1)$$

This result, derived in an accelerated coordinate system[1] in which the detector is at rest, has generally remained somewhat puzzling, especially with regard to the change in the state of the field which accompanies the excitation of the detector. Some light was cast on the subject by Unruh and Wald[2] in a study of the process in an inertial frame of reference (i.e., in Minkowski space-time). In such a frame of reference, the excitation of the detector (which might be, e.g., a system with two levels, 1 and 2) is accompanied by the emission of a quantum of the corresponding field with which the detector is interacting (this important circumstance was either ignored or not understood in several earlier studies). For simplicity, a scalar field was discussed in Refs. 1 and 2 and in several other papers. However, the same result is found[3–5] for the case of an electromagnetic field, and a photon is emitted when the detector is excited (when, say, there is a transition of the detector from its lower level, with energy ϵ_l, to a level with an energy $\epsilon_t > \epsilon_l$). It is this emission of a photon that will be discussed below, for definiteness.

The complexity of the calculations of Refs. 1 and 2 and also the use of "classical electromagnetic zero-point radiation"[3–5] leave somewhat unclear the nature and physical mechanism for the excitation and emission of the accelerated detector. In this connection, we feel it useful to call attention to the circumstance that an analogous excitation occurs in a situation of which we have been aware for a long time now,[6,7]: the anomalous Doppler effect.

We assume that a detector with two levels is moving at a constant velocity $\mathbf{v}$ through a medium with a refractive index $n(\omega)$. The energies of the detector in the states l and t are $E_{l,t} = [(m_0 + m_{l,t})^2 c^4 + p^2 c^2]^{1/2}$; $\epsilon_{l,t} = m_{l,t} c^2$; the energy and momentum of the emitted photon are $\hbar\omega$ and $\hbar\mathbf{k}$; and $k = (\omega/c) n(\omega)$. For simplicity, we write the energy and momentum conservation laws during the emission of the photon for the case in which we have $m_{l,t} \ll m_0$ and in which the recoil is slight (see Ref. 6 for the general case):

$$\mathbf{v}\Delta\mathbf{p} + \Delta\epsilon\sqrt{1 - v^2/c^2} + \hbar\omega = 0, \quad \hbar\mathbf{k} + \Delta\mathbf{p} = 0, \tag{2}$$

where $\Delta\mathbf{p} = \mathbf{p}_2 - \mathbf{p}_1$ is the change in the momentum of the detector upon the emission, $\mathbf{v}\Delta\mathbf{p}$ is the change in its kinetic energy, and $\Delta\epsilon = \epsilon_2 - \epsilon_1$ is the change in the energy in the rest frame of the detector. The indices 1 and 2 correspond to the states before and after the emission. From (2) we find

$$\Delta\epsilon\sqrt{1 - v^2/c^2} = -\hbar\omega(1 - (vn/c)\cos\theta), \quad \mathbf{kv} = kv\cos\theta. \tag{3}$$

In the region of the normal Doppler effect, with $(vn/c)\cos\theta < 1$, we have $\Delta\epsilon = \epsilon_2 - \epsilon_1 < 0$ and thus $\epsilon_1 = \epsilon_t$ and $\epsilon_2 = \epsilon_l < \epsilon_t$; i.e., the emission is accompanied by a transition from the upper state t to the lower state l. In contrast, in the region of the anomalous Doppler effect, with $(vn/c)\cos\theta > 1$ [i.e., for an angle $\theta < \theta_0$, where $\cos\theta_0 = c/nv$, and θ_0 is the angle corresponding to (Vavilov-) Čerenkov radiation], we have $\Delta\epsilon > 0$, $\epsilon_1 = \epsilon_l$, and $\epsilon_2 = \epsilon_t > \epsilon_l$, so that the emission of the photon is accompanied by the excitation of the detector. We thus see that expression (3) is identical to the general formula for the Doppler effect in a medium:

$$\omega = \frac{\omega_{00}\sqrt{1 - v^2/c^2}}{|1 - (v/c)n(\omega)\cos\theta|}, \quad \omega_{00} = \frac{\epsilon_t - \epsilon_l}{\hbar}. \tag{4}$$

Methodologically, the introduction of a medium allows us to examine velocities above the velocity of light $[v > c/n(\omega)]$ and thus the anomalous Doppler effect, which does not occur at velocities below the velocity of light and which therefore never occurs in a vacuum at $v < c$ (we are obliged to refer the reader to Ref. 7, and the bibliography there, for the fine points, especially those dealing with a possible realization of motion at a velocity above the velocity of light).

We thus have a detector moving at a velocity above the velocity of light which is excited in a process that involves the emission of photons, even if it is initially in its lowest (ground) level. In the steady state, the populations of levels l and t (or, in general, of any levels of a detector) are determined by the emission probabilities in the regions of the normal and anomalous Doppler effects (see Ref. 7, the bibliography cited there, and Ref. 8). In hydrodynamics we have an analogous situation, where the role of motion above the velocity of light is played by motion at a supersonic velocity.[9,10]

In a vacuum, as a source moves at a constant velocity $v \leq c$ (this restriction is by no means a trivial one,[7] but we will use it here), there is no Čerenkov radiation, and the anomalous Doppler effect cannot occur. The detector is therefore not excited. In the case of accelerated motion, in contrast, a charge of course begins to emit even in a vacuum, and a detector—a system with internal degrees of freedom (an oscillator, an atom, etc.)—can be excited. We see no fundamental distinction here with the case of the anomalous Doppler effect. Energy and momentum are of course conserved for the detector and the radiation, but with allowance for the forces that accelerate the detector. The distribution of the detector in its levels (i.e., the populations of the energy levels of the detector) depends on the nature of the motion of the detector and on the probabilities for the corresponding transitions.[11] In this regard, motion at a constant acceleration is a special case: The steady-state level distribution turns out to be a thermal distribution[1–5,11,12] with the temperature in (1). During motion at a constant velocity above the velocity of light in a medium, i.e., under the conditions of the anomalous Doppler effect, the absence of anything in the way of a universal excitation temperature of the detector is clear even from dimensionality considerations (on the other hand, in certain particular cases the level distribution may be thermal with a temperature determined by the spacing of levels[8]). In contrast, during accelerated motion the combination $\hbar a/kc$ has the dimensionality of a temperature [see (1); the characteristic frequency is $\omega \sim 2\pi a/c$]. The very fact that the distribution in levels of the detector under the condition $\mathbf{a} = \text{const}$ is a thermal distribution stems from the equivalence principle, according to which an accelerated frame of reference with $\mathbf{a} = \text{const}$ is locally equivalent to a frame of reference which is at rest in a static, uniform, gravitational field $\mathbf{g} = -\mathbf{a}$. In such a gravitational field, a thermodynamic equilibrium prevails with an ordinary thermal distribution. We know that an ordinary thermal distribution with the temperature (1), where $a = GM/r_g^2$, $r_g = 2GM/c^2$, also arises in the theory of black holes (M is the mass of the hole). We should therefore expect that in a system in uniform acceleration there would also be a thermal distribution of the detector in its levels. Since we do not yet have a clear understanding of the form of the equivalence principle in the quantum-mechanical case, we wish to emphasize that the result in (1) has also been derived[1–4,11–13] without any use of the equivalence principle.

In the approximation corresponding to Eqs. (2) and (3), the Doppler effect is classical [expression (2) has been written, for simplicity, in quantum-mechanical terms, with the use of Planck's constant $\hbar$, but this approach could be avoided[7]]. Spontaneous emission and induced emission also occur in the classical theory.[14] If, on the other hand, we are concerned with the excitation of a detector with discrete energy levels or a detector in its ground state in a vacuum, we need to take a quantum approach (here we are not concerned with the possibility of introducing a classical zero-point field[3–5]; that approach seems to us to be no more than methodological). If we thus have a classical oscillator, and if its oscillation amplitude is zero, it will not be driven into oscillation (excited) either in the anomalous Doppler effect or during acceleration of the oscillator as a whole. In contrast, a quantum-mechanical oscillator, an atom, or whatever in its ground state, interacting with electromagnetic or other quantized fields will generally be excited both in the anomalous Doppler effect and during acceleration. We are dealing here with a quantum-mechanical effect [hence Planck's constant $\hbar$ in (1)]. We also note that the emission of a neutral polarized source in uniform motion at a velocity above the velocity of light, analyzed in Ref. 15, is a Doppler radiation under conditions such that the source is undergoing zero-point oscillations (or, more precisely, under conditions such that, say, zero-point oscillations of the electric polarization are occurring in the source). The detector is excited not only during acceleration or at a velocity above the velocity of light but also at a velocity below the velocity of light, if it is moving through an inhomogeneous or time-varying medium (here we are talking about analogs of transition radiation[7]). In addition to the excitation and emission of the detectors, one could study the quantum emission of mirrors and, more generally, of polarizable media (Ref. 16, for example). Such problems deserve a more detailed analysis. The same can be said of some of the other questions mentioned here. It is necessary to analyze the change in the momentum and mass of an accelerated source (detector), with allowance for its excitation and emission,[17] and to calculate the reaction of radiation in a medium not only in the case of uniform motion[7,18] but also for accelerated motion.

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