Drift-dissipative excitation of electron vortices in a plasma

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(Submitted 27 December 1985)
Pis'ma Zh. Eksp. Teor. Fiz. **43**, No. 6, 268–270 (25 March 1986)

Vortex tubes with a diameter smaller than the ion Larmor radius are generated in an inhomogeneous plasma as a result of a dissipation involving electrons. The mixing in these vortices may be the primary mechanism for the anomalous electron thermal conductivity in a plasma with $m/M \ll \beta \ll 1$.

The existing theory of anomalous transport in plasmas[1,2] predicts that suprathermal fluctuations of magnitude on the order of the skin length are primarily responsible for the electron thermal conductivity. The reason is that at such a scale the electrons are no longer frozen in the magnetic field. In a magnetic field with shear, in both the linear and weakly turbulent pictures, we find a problem involving the localization of wave packets. In the present letter we show that nonlinear effects in a plasma can easily give rise to solitary structure in the form of electron vortices which are elongated along the magnetic field and which have a transverse dimension less than the ion Larmor radius r_{Bi}. These structures are shown to be insensitive to a shear. Such vortices move at a velocity lower than the drift velocity, so that their amplitude may be increased by Landau damping or by a collisional dissipation involving electrons. This phenomenon is analogous to the linear drift-dissipative instability of slow electrostatic drift wave with a length greater than r_{Bi}. Analogous vortices with a dimension much greater than r_{Bi} were found in Refs. 4 and 5. In them, the ions are described by hydrodynamic equations. Since the size of the vortices under consideration here is much less r_{Bi}, and the frequency is much less than ω_{Bi}, the ion density in the electric potential ϕ of the vortex has a Boltzmann distribution[3]:

$$n = n_0(1 + \kappa x - \tau\phi); \quad \tau \equiv T_e/T_i; \qquad n_0, \kappa = \text{const}. \tag{1}$$

According to (1), the electron can be described by the following system of dimensionless equations:

$$\frac{dN}{dt} = \frac{d\Delta A}{dz}; \quad \frac{d}{dt} \equiv \frac{\partial}{\partial t} + \{\phi, ...\}; \quad \frac{d}{dz} \equiv \frac{\partial}{\partial z} - \{A, ...\} \tag{2}$$

$$(1+\nu)E_{\parallel} = \frac{dN}{dz}; \quad \{\phi, N\} = \frac{\partial\phi}{\partial x}\frac{\partial N}{\partial y} - \frac{\partial\phi}{\partial y}\frac{\partial N}{\partial x}, \tag{3}$$

where (2) is the electron-continuity equation, and (3) is the equation of motion of the electrons along the magnetic field. In the case under consideration here, with the wave velocity along z being much lower than v_{Te}, Eq. (3) reduces to the balance equation of the gradient pressure of the electrons and of the longitudinal electric field $E_{\parallel} = -d\phi/dz - \partial A/\partial t$. By analogy with Ref. 5, we transform to dimensionless variables:

0021-3640/86/060343-03$01.00

$$\omega_{Bi}t \to t; \quad e\varphi/T_e \to \phi; \quad A_z e c_A/cT_e \to A; \quad \frac{n}{n_0} \to N$$

$$\kappa c_S/\omega_{Bi} \to \kappa; \quad (x,y)\omega_{Bi}/c_S \to (x,y); \quad z\omega_{Bi}/c_A \to z$$

Here A_z is the component of the vector potential along the static magnetic field, c_A is the Alfvén velocity, c_S is the ion acoustic velocity, and $\Delta \equiv \partial^2/\partial x^2 + \partial^2/\partial y^2$. The operator ν in (3) describes a dissipation by electrons in the linear approximation. Comparing with the linear theory,[3] we easily see that the Fourier spectrum of this operator is

$$\nu_{\mathbf{k},\omega} = i\sqrt{\pi}(\omega + \kappa k_y)/|k_z|v_{Te}. \tag{4}$$

Making use of quasineutrality, we substitute (1) into (3) and (2). Ignoring dissipation for the moment, we find a system of equations for small-scale drift waves:

$$\tau\partial\phi/\partial t + \kappa\partial\phi/\partial y = d\Delta A/dz \tag{5}$$

$$\partial A/\partial t - \kappa\partial A/\partial y = -(1+\tau)d\phi/dz. \tag{6}$$

We seek a steady-state two-dimensional solution of (5) and (6), which is traveling along y at a velocity u and which is tilted at an angle α. We then find from (6)

$$\phi(x,\eta) = \frac{u+\kappa}{\alpha(1+\tau)}A(x,\eta); \quad \eta = y + \alpha z - ut. \tag{7}$$

The coefficient in (7) is found from the condition that ϕ and A decay at infinity. We can thus rewrite (5) as

$$\Delta A + \alpha s^2 x = f(A - \alpha x); \quad s^2 = \frac{(\kappa+u)(\kappa-\tau u)}{\alpha^2(1+\tau)}, \tag{8}$$

where f is an arbitrary function. We solve Eq. (8) by the Larichev-Reznik method.[6,7] If (8) is to have a localized solution, we must have $s^2>0$; this condition is met if the propagation velocity lies in the interval between the ion drift velocity κ/τ and the electron drift velocity κ. We introduce the coordinate $r^2 = x^2 + \eta^2$, and we write f as a linear function with different coefficients inside and outside a circle of radius $r_0 \ll r_{Bi}$. The coefficients are chosen in such a way that we find a localized solution of Eq. (8) in the form

$$A = b_0F_0(r) + \alpha r_0F_1(r)x/r. \tag{9}$$

Here F_0 and F_1 are functions which are, along with their first derivatives, continuous and which are expressed in terms of Bessel functions inside the circle and modified Hankel functions outside it.[6,7] At the joining boundary, $r = r_0$, we must set $A - \alpha x = \text{const}$. The amplitude b_0, the angle α, the radius r_0, and the velocity u remain arbitrary. Solution (9) decays as $\exp(-sr)$ at infinity.

We now examine the effect of a dissipation on this solution. We note that system (5), (6) conserves the integral

$$W = \int[(\nabla_\perp A)^2 + \tau(1+\tau)\phi^2]dxdydz. \tag{10}$$

When dissipation is taken into account in this form, as in (3), this integral varies over time:

$$\partial W/\partial t = -\int \Delta A\, \nu E_{\parallel}\, dxdydz. \tag{11}$$

Under the assumption that dissipation is slight, we can substitute solution (9) and (4) into (11). Switching to dimensionless variables in (4), and noting that in a steady-state vortex we have $\omega = uk_y$, according to (7), we find

$$\partial W/\partial t = |\alpha| s^2 c_A v_{Te}^{-1} \int k^2 |k_y| A_{\mathbf{k}}^2 d\mathbf{k}. \tag{12}$$

We thus see that W increases over time. The amplitude b_0 may increase without a change in r_0, α, or u. As the amplitude increases, a plateau forms, and the Landau damping becomes a weakly collisional dissipation.[8] This circumstance slows the growth of the vortices slightly. The growth will apparently continue until the approximations used in deriving the original equations are violated, i.e., until we have $\phi \sim 1$, etc.

In summary, we have shown that when the nonlinearity is taken into account packets of drift-Alfvén waves of size less than r_{Bi} form vortex tubes (electron vortices), which propagate at a velocity below the drift velocity. As a result, the dissipation by electrons leads to a buildup of these vortices to energy densities comparable to the thermal energy density. The transport coefficients in the presence of such fluctuations were found in Ref. 2.

We wish to thank O. P. Pogutse for useful advice and discussions.

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Translated by Dave Parsons