MHD plasma stabilization in axially symmetric open confinement system

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(Submitted 27 January 1986)
Pis'ma Zh. Eksp. Teor. Fiz. **43**, No. 6, 270–272 (25 March 1986)

It is shown that MHD stability in low-pressure plasma can be achieved in simple confinement systems by connecting them to a thin confinement system with a curvature of opposite sign.

1. In axially symmetric confinement systems of highly simplified design (in which the working magnetic flux is confined between the coils and the axis) the plasma with a natural distribution of pressure which decreases toward the magnetic mirrors is subjected to a magnetohydrodynamic flute instability.[1] An example of a mirror system, which is stable with respect to large-scale flute perturbations, was recently reported in Ref. 2. This mirror system shows that there are confinement systems of the type described above, in which the stability of all modes can be achieved upon establishing a connection with an element with an opposite curvature sign of the lines of force. In this letter we will identify the confinement systems that have this property. For maximum simplicity, we consider a case in which the anisotropy is strong and the transverse pressure $p_\perp$ is considerably higher than the longitudinal pressure $p_\parallel$ (low mirror ratio or a disk-shaped plasma near the field minimum).

2. The magnetic configuration is shown in Fig. 1. We assume that confinement systems 1 and 2 are linked by a plasma whose pressure is negligible but whose conductivity is high, so that the flute perturbation extends along the entire line of force of the plasma. We also assume that 1) cell 2 is thin: $|\partial \ln B / \partial \ln p| \ll 1$ (the cell is situated far from the axis and/or has a strong field) and 2) cell 1 has a strong anisotropy and the relative variation of $\partial B / \partial \psi$ along the field (where ψ is the flux coordinate reckoned from the axis) is less than, or on the order of, $p_\parallel / p_\perp$ (this is possible if, for example, the given confinement system has an equatorial symmetry plane; note that we have so

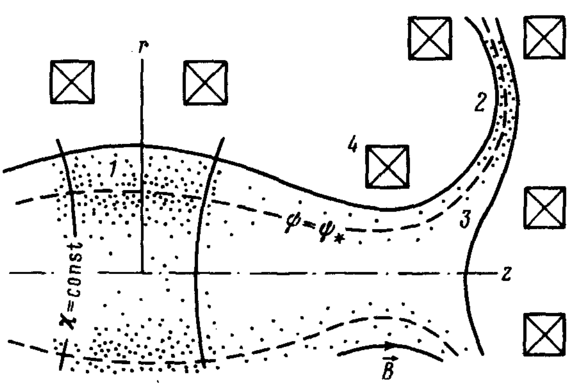

FIG. 1. Schematic of the configuration. 1—Principal confinement; 2—compensating confinement; 3—connecting plasma; 4—current coils.

0021-3640/86/060346-03$01.00

far not required that the confinement system be of the simplest kind). We can then write the Kruskal-Oberman[3] criterion for MHD stability for a low-pressure plasma ($\beta = 8\pi(p_\perp + p_\parallel)/B^2 \ll 1$) in the form[4]

$$W_1 + W_2 > 0, \tag{1}$$

where

$$W_1 = \int \left[-\frac{\partial}{\partial\psi}\left(\frac{p_\perp + p_\parallel}{B^2}\right)\frac{\partial B}{\partial\psi} + \frac{p_\perp - p_\parallel}{B^3}\left(\frac{\partial B}{\partial\psi}\right)^2 \right]\frac{d\chi}{B}, \tag{2}$$

$$W_2 = -\int \frac{\partial(p_\perp + p_\parallel)}{\partial\psi}\frac{\partial B}{\partial\psi}\frac{d\chi}{B^3}. \tag{3}$$

Here χ is the longitudinal coordinate ($\vec{\nabla}\chi = \mathbf{B}$), and the integration in $W_{1,2}$ is carried out over regions 1 and 2, respectively. Introducing $\alpha(0<\alpha<1)$ such that $(p_\perp - p_\parallel)_1 \geqslant \alpha(p_\perp + p_\parallel)_1$, we find from (1) the sufficient condition for the stability

$$W = -\int \frac{\partial}{\partial\psi}\left(\frac{p}{B^{2+\alpha}}\right)\frac{\partial B}{\partial\psi}\frac{d\chi}{B^{1-\alpha}} > 0. \tag{4}$$

where $p = p_\perp + p_\parallel$; the integration is carried out along the entire system, and in region 2 only $\partial p/\partial\psi$ is important in the combination $\partial(p/B^{2+\alpha})/\partial\psi$. In the limit $\alpha \to 0$ [i.e., ignoring the second positive term in (3)], expression (4) becomes the sufficient condition given in Ref. 4.

Let us assume that the plasma occupies a layer near the surface, $\psi = \psi_*$, in which[1)] $(\psi - \psi_*)\partial(p/B^{2+\alpha})/\partial\psi \leqslant 0$. In this case, expression (4) is satisfied if the pressure ratio in confinement systems 1 and 2 is such that

$$\int \left(\frac{1}{B^{1-\alpha}}\frac{\partial B}{\partial\psi}\right)_{\psi=\psi_*}\frac{p}{B^{2+\alpha}}\,d\chi = 0 \tag{5}$$

along all lines of force that pass through the plasma and

$$\frac{\partial}{\partial\psi}\left(\frac{1}{B^{1-\alpha}}\frac{\partial B}{\partial\psi}\right) > 0 \tag{6}$$

in a given layer in confinement system 1. If the plasma in element 1 is situated near the equatorial symmetry plane, we can write (6) in the form

$$\frac{\partial}{\partial r}\left(\frac{1}{B^{2-\alpha}r}\frac{\partial B}{\partial r}\right) > 0. \tag{7}$$

Condition (6) can be expressed in terms of the characteristics of the line of force:

$$\frac{d''}{d} + \frac{k}{r}\cos\theta + \alpha k^2 > 0. \tag{8}$$

Here $r(s)$ is the distance from the axis, $k(s)$ is the curvature, $\theta(s)$ is the angle between the normal to the surface $\psi =$ const and the radial direction, s is the length along the line of force, $d = r^{-1}(s)B^{-1}(s)$ is the thickness of the magnetic tube of force with a unit flux, and the prime is the derivative of s. The individual terms on the left side of (8) can be treated in a straightforward manner. In the case of cancellation of the average curvatures [expression (5)], the stabilization because of $d'' > 0$ stems from the action of the "hidden" (against the background of the overall curvature) concavity of the tube of force. The origin of $k \cos\theta / r$ is traceable to the fact that $\partial B/\partial\psi = -k/r$ in (6) contains not only the curvature but also the radial coordinate, so that in the case of ψ-independent curvature the derivative $\partial B/\partial\psi$ will increase with increasing ψ, since r will increase with increasing ψ. The term quadratic in k describes the effect of the large curvature of the magnetic field.

3. Let us define concretely condition (6) for the case we are considering here, in which cell 1 is a very simple confinement system (see Sec. 1). The vector field potential of such a confinement system is determined by a single function $b(z)$—the field along the axis

$$A_\varphi = \frac{b(z)}{2} r - \frac{b''}{16} r^3 + \frac{b^{\mathrm{IV}}}{384} r^5 - \dots . \tag{9}$$

For longitudinal confinement, the function $b(z)$ must have a minimum between the mirrors, say, at $z = 0$. For simplicity, we consider the case of the equatorial symmetry and we restrict the analysis to the terms of the expansion in (9) that were written down. We can then reduce (7) to

$$b^{\mathrm{IV}}(0) > 4\left(1 - \frac{\alpha}{2}\right) \frac{(b''(0))^2}{b(0)} . \tag{10}$$

According to (10), stabilization does not require the curvature to be large, so that it can be achieved even in the region around the axis. Condition (6) for $B^{\alpha-1}\partial B/\partial\psi$ is not as rigorous as the inequality $\partial(\psi B^{\alpha-1}\partial B/\partial\psi)/\partial\psi > 0$, the satisfaction of which would result in the suppression (as in the case of Ref. 2) of the first mode in a system with anisotropic plasma in which there is no compensating confinement system 2.

If the overall curvature cancels out, i.e., if the contribution to W from the ψ-independent leading term in $\partial B/\partial\psi$ vanishes, then inequality (10) and, in general, inequality (6), account for the mean magnetic well, since the derivative $\partial^2 B/\partial\psi^2 > (1-\alpha)B^{-1}(\partial B/\partial\psi)^2$ is positive.

[1)]We should point out that profiles with a nonzero pressure along the axis are also valid.

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Translated by S. J. Amoretty