# The Josephson effect in superconductors with heavy fermions

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The dependence of the Josephson current that flows between an ordinary and extraordinary superconductor on the temperature and on the angle between the surface of the contact and the crystal axes is determined.

Experimental data on the low-temperature behavior of the heat capacity, on NMR, and on ultrasound absorption in superconductors with heavy fermions ($CeCu_2Si_2$, $UBe_{13}$, $UPt_3$) indicate that the pairing of electrons in these compounds is unusual and that the order parameter transforms according to a nontrivial symmetry-group representation of the crystal. The Josephson effect occurring between these superconductors and ordinary superconductors was studied experimentally by Steglich *et al.*[1]

A steady tunneling at approximately the same level as in ordinary superconductors was observed in a $CeCu_2Si_2$ polycrystal. The Josephson effect was not detected in a $UPt_3$ single crystal.

In this letter we show that the critical current between an ordinary and extraor-

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dinary superconductor is strongly dependent on the angle between the surface of the sample and the crystal axes and that it vanishes in certain directions. The angular dependence can be determined phenomenologically from the symmetry considerations. The free energy of the contact must be a scalar quantity that depends on the vector directed normal to the surface and it must linearly depend in first order in transparency on the order parameter in the extraordinary superconductor.

Let us first consider the case in which the nontrivial order parameter is a pseudoscalar, i.e., the case in which it remains constant during rotation and changes sign upon inversion. The free energy of the contact and hence the critical current are proportional to the pseudoscalar which consists of a unit normal vector $\mathbf{n}$. For crystal of cubic symmetry such a pseudoscalar can be written in a very simple form:

$$I(\varphi) = A n_x n_y n_z (n_x^2 - n_y^2)(n_y^2 - n_z^2)(n_z^2 - n_x^2) \sin\varphi, \tag{1}$$

where $n_x$, $n_y$, and $n_z$ are the projections of the normal vector onto the crystal axes, and $\varphi$ is the phase difference. The critical current vanishes when the normal vector lies in a symmetry plane of the cube. The disappearance of the current follows from the symmetry considerations and is not related in any way to the specific form of Eq. (1), since $\mathbf{n}$ does not change upon reflection in this plane and the order parameter changes sign.

To estimate the value of the coefficient $A$ and its temperature dependence, we write the miscroscopic formula for the current that flows through the contact

$$I(\varphi) = 2e \operatorname{Im} \Sigma F^*_{\alpha\beta}(\mathbf{p}) T_{\beta\mu}(\mathbf{p},\mathbf{k}) K(\mathbf{x}) \Delta_{\mu\nu}(\mathbf{k},\mathbf{x}) T_{\alpha\nu}(-\mathbf{p},-\mathbf{k}), \tag{2}$$

where $F^*$ is the Gor'kov function in the ordinary superconductor, $T_{\beta\mu}(\mathbf{p},\mathbf{k})$ are the matrix elements of the tunnel Hamiltonian, and $K(x)$ is an ordinary kernel in an integral equation which links the Gor'kov function in the extraordinary superconductor with the order parameter $\Delta_{\mu\nu}$.

In crystal with an inversion center, the single-electron wave functions are characterized by a quasimomentum $\mathbf{k}$ and a pseudospin $\mu$ which describes double degeneracy of the level. The pseudospin coincides with the spin in the absence of spin-orbit interaction. The spin-orbit interaction accounts for the fact that the eigenstate is a superposition of the states with different spin projections

$$|\mathbf{k}\mu\rangle = e^{i(\mathbf{kr})} \sum_{\mathbf{d}} [u_{\mathbf{k}}(\mathbf{r}-\mathbf{d}) + (\mathbf{u}^1_{\mathbf{k}}(\mathbf{r}-\mathbf{d})\sigma_{\mu\alpha})]\,|\alpha\rangle \cdot \tag{3}$$

In (3) the sum is over the crystal lattice, where $u_{\mathbf{k}}(\mathbf{r})$ is the scalar function, and $\mathbf{u}^1_{\mathbf{k}}(\mathbf{r})$ is the pseudovector function which in the simplest case is given by

$$\mathbf{u}^1_{\mathbf{k}}(\mathbf{r}) = [\mathbf{kr}]\, u_1. \tag{4}$$

In such crystals the superconducting states are characterized by their particular parity[2,3]

$$\Delta^g(\mathbf{k}) = \psi(\mathbf{k}) i\sigma^y; \;\; \Delta^u(\mathbf{k}) = (\vec{\sigma}\mathbf{d}(\mathbf{k}) i\sigma^y. \tag{5}$$

If there is no spin-orbit interaction, then $T_{\alpha\beta}$ is a diagonal matrix and there is no Josephson effect between the even and the odd superconductors.[4]

Fenton[5] pointed out that the spin-orbit interaction accounts for the nonvanishing Josephson current. He ignored, however, the crystal structure, and the effect he obtained was small compared with the usual effect, at least because of the small ratio of the interatomic distances to the size of the pair, $\xi$. The ratio is small because the order parameter varies slowly near the surface of the sample. A considerably stronger effect occurs as a result of the nondiagonal nature of the tunnel Hamiltonian $T_{\alpha\nu}$. For the tunnel Hamiltonian to be nondiagonal, the spin-orbit interaction need not necessarily occur in the insulating layer: the insulating layer need only be present in the superconductor. This effect, which is similar to the spin-orbit scattering by light impurities in metals with heavy atoms, is associated with the fact that the index $\nu$ describes the projection of the pseudospin, rather than the spin. In a microscopic calculation of the tunnel Hamiltonian $T_{\alpha\nu}$, the wave functions in the insulator should be matched with the wave functions of the type in (3), which describe the behavior of an electron in a metal with heavy atoms. Since the heavy atoms are found only on one side of the boundary, the mean vector **r** in expression (4) is directed along the vector **n**. As a result, we have

$$T_{\alpha\nu}(\mathbf{k}) = T(\delta_{\alpha\nu} + \lambda/k_F([\mathbf{kn}]\vec{\sigma}_{\alpha\nu})), \qquad (6)$$

where the dimensionless parameter $\lambda$ in a metal with heavy atoms is on the order of unity. Substituting expression (5) and (6) into expression (2), we find the following expression for the odd pairing:

$$I(\varphi) = 2e\,\mathrm{Im}\int \lambda T^2 F^* K(\mathbf{x})([\mathbf{d}(\mathbf{k},\mathbf{x})\,\mathbf{k}]\mathbf{n})\,d\mathbf{x}\,d\mathbf{k}. \qquad (7)$$

The **k** dependence of the order parameter **d** for different representations of several groups was found in Refs. 2 and 3. In the integration of expression (7) over the angles of the vector **k**, one should take into account that $T^2$ and $K(\mathbf{x})$, and also $\mathbf{d}(\mathbf{k},\mathbf{x})$ for representations other than the one-dimensional, are complex functions of $(\mathbf{kn})$. As a result, we find expression (1) for the pseudoscalar representation $A_1$ of the cubic group. At low temperatures the constant $A$ differs by only a factor on the order of unity from the standard expression for the current that passes through the contact. This factor is associated with the spin-orbit interaction and the anisotropy. An additional small parameter on the order of $\xi_0/\xi(T)$ appears near $T_c$ because the order parameter of the uncommon pairing falls off as the surface is approached within $x \sim \xi(T)$, while the kernel $K(\mathbf{x})$ decreases at a distance of $\xi_0$. The parameter $A$ is of the same order of magnitude for the other phases.

The dependence of the Josephson current on the angles of the vector **n** for the other phases can be found from Eqs. (2) and (7) and from the symmetry considerations. The spin-orbit interaction is unimportant for the even phases. The dependence $I(\mathbf{n})$ is the same as the dependence of the scalar order parameter $\psi(\mathbf{k})$ on the angles of the momentum **k**. The current vanishes when the normal to the surface runs in the direction along which the gap in the excitation spectrum vanishes. These directions have been determined in Ref. 2.

For odd phases the current vanishes in two cases: 1) if the symmetry axis in the plane of the contact is such that a 180° rotation around this axis does not change the order parameter and 2) if the normal to the surface is directed along the axis the rotation around which results in the multiplication of the order parameter by the phase factor. The symmetry classes of the order parameter are listed in Ref. 2. A highly simplified angular dependence of the Josephson current which satisfies these requirements is given by

$$I(\varphi) = A\,\mathrm{Im} f(\mathbf{n}) e^{i\varphi},$$

where $f(\mathbf{n})$ for different representations is:

cubic group ($UBe_{13}$)

$A_1: n_x n_y n_z (n_x^2 - n_y^2)(n_y^2 - n_z^2)(n_z^2 - n_x^2)$ ; $A_2: n_x n_y n_z$ ;

$E: n_x n_y n_z (\eta_1 (n_x^2 + e^{-2\pi i/3} n_y^2 + e^{2\pi i/3} n_z^2) - \eta_2 (n_x^2 + e^{2\pi i/3} n_y^2 + e^{-2\pi i/3} n_z^2))$ ;

$F_2: \eta_1 n_x (n_y^2 - n_z^2) + \eta_2 n_y (n_z^2 - n_x^2) + \eta_3 n_z (n_x^2 - n_y^2)$ ; $F_1: \eta_1 n_x + \eta_2 n_y + \eta_3 n_z$ ;

tetragonal group ($CeCu_2Si_2$)

$A_1: n_x n_y n_z (n_x^2 - n_y^2)$ ; $A_2: n_z$ ;

$B_1: n_x n_y n_z$ ; $B_2: n_z (n_x^2 - n_y^2)$ ;

$E: \eta_1 n_y - \eta_2 n_x$ ;

hexagonal group ($UPt_3$)

$A_1: n_z (n_x^3 - 3n_x n_y^2)(n_y^3 - 3n_y n_x^2)$ ; $A_2: n_z$ ;

$B_1: n_x^3 - 3n_x n_y^2$ ; $B_2: n_y^3 - 3n_y n_x^2$ ;

$E_1: \eta_1 n_y - \eta_2 n_x$ ;

$E_2: \eta_1 n_z (n_x - i n_y)^2 - \eta_2 n_z (n_x + i n_y)^2$.

For other than the one-dimensional representations, the possible values of the parameters $\eta_i$ are given in Ref. 2. For some phases these values are ambiguous. In this case, domain walls can exist in the bulk of the crystal. As to whether these domain walls interact with the surface requires further study. If we are dealing with a magnetic phase and a complex order parameter, we will have $I(\varphi) = I_1 \cos\varphi + I_2 \sin\varphi$, where $I_c^2 = I_1^2 + I_2^2$.

The minimum energy of the contact does not always correspond to the point $\varphi = 0$. If, for example, a plate with an odd phase between two contacts is inserted into a superconducting circuit, the vector **n** and hence the energy of these contacts will have opposite signs. If, for example, a plate with an odd phase between two contacts is inserted into a superconducting circuit, the vector **n** and hence the energy of these contacts will have opposite signs. If the minimum energy of one contact corresponds to the phase difference $\varphi = 0$, the minimum energy of the other contact will correspond to the phase difference $\varphi = \pi$. This circuit will therefore have a half-integer number of fluxoids.

The absence of this effect in a $UPt_3$ single crystal may conceivably stem from such positioning of the contact plane relative to the crystal axes at which the Josephson current vanishes. The presence of a current in the Josephson junction of an ordinary superconductor with $CeCuSi_2$ does not mean that there is an ordinary pairing in this contact

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