Buildup of neutrino oscillations in the Earth

V. K. Ermilova, V. A. Tsarev, and V. A. Chechin
P. N. Lebedev Physics Institute, Academy of Sciences of the USSR

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The neutrino oscillations in the Earth can be built up and the sensitivity of the experiments on the search for neutrino oscillations (at $\theta_\upsilon \ll 1$) and neutrino probing of the Earth can therefore be increased.

The coherent scattering of neutrinos in matter over a distance $l_0 = 1/2G N_e \cong 3.5 \times 10^4$ km$/\rho$ (G is the Fermi constant, N_e is the electron density, and ρ is the density of the substance in g/cm^3) may have an appreciable effect on the neutrino oscillations.[1] For example, in underground experiments with atmospheric neutrinos that pass through the Earth, the coherent scattering of neutrinos reduces by approximately an order of magnitude their sensitivity to[2] Δm^2. At the same time, the oscillations may increase under certain "resonance" conditions at $\theta_\upsilon \ll 1$, where θ_υ is the mixing angle in a vacuum. In the present letter we show that such a buildup of oscillations can occur in neutrino beams from accelerators ($1\ \text{GeV} \lesssim E_\nu \lesssim 1$ TeV) when they pass through the Earth. This effect may turn out to be useful in the search for these oscillations and in the neutrino geophysics.[4]

Since the distribution of matter $\rho(x)$ in the Earth is highly nonuniform, the solution obtained in Ref. 1 cannot be used. In the same case of two types of neutrinos (ν_e and ν_μ), we can switch in the equations describing the oscillations[1] to the original basis of ν_e and ν_μ. We then would have

$$i\frac{d}{d\tau}\left|\begin{matrix}\nu_\mu\\ \nu_e\end{matrix}\right\rangle = \hat{H}(\tau)\left|\begin{matrix}\nu_\mu\\ \nu_e\end{matrix}\right\rangle, \tag{1}$$

where $\hat{H}(\tau) = \boldsymbol{\omega}(\tau)\boldsymbol{\sigma}$ ($\boldsymbol{\sigma}$ are the Pauli matrices),

$$\boldsymbol{\omega}(\tau) = \{-\sin 2\theta_\upsilon, 0, \cos 2\theta_\upsilon - l_\upsilon/l_0(\tau)\},$$

$$l_\upsilon = 4\pi E_\nu/(m_1^2 - m_2^2) \cong 2.5\ \text{km}\, E_\nu(\text{GeV})/\Delta m^2(\text{eV}^2)$$

is the length of the vacuum oscillations, $\tau = \pi x/l_\upsilon$, and $\hbar = c = 1$. Consequently,

$$\left|\begin{matrix}\nu_\mu(\tau)\\ \nu_e(\tau)\end{matrix}\right\rangle = \hat{T}_\tau \exp\left[-i\int_0^\tau \hat{H}(\tau')d\tau'\right]\left|\begin{matrix}\nu_\mu(0)\\ \nu_e(0)\end{matrix}\right\rangle. \tag{2}$$

In the case $\rho = \text{const} = \bar{\rho}$ we find Wolfenstein's[1] result:

$$P_{\nu_\mu \to \nu_e}(x) = (\sin^2 2\theta_\upsilon)\omega^{-2}\sin^2 \omega\tau;\ \omega = \left(1 - 2\cos 2\theta_\upsilon \frac{l_\upsilon}{\bar{l}_0} + \frac{l_\upsilon^2}{\bar{l}_0^2}\right)^{1/2}.$$

For a variable density we can switch, after eliminating $|\nu_\mu\rangle$, to a second-order equation for $\xi = |\nu_e\rangle$

$$\frac{d^2\xi}{d\tau^2} + f(\tau)\xi = 0;\ f(\tau) = \omega^2(\tau) + iD(\tau);\ D(\tau) = \frac{d}{d\tau}\left[\frac{l_\nu}{l_0(\tau)}\right], \tag{3}$$

with the initial conditions $\xi(0) = 0$ and $(d/d\tau)\xi(0) = i\sin 2\theta_\nu$. The other method of calculating $P_{\nu_\mu\to\nu_e}$ is the use in Ref. 1 of the Heisenberg equation of motion for the spin operator $\mathbf{s} = \boldsymbol{\sigma}/2$:

$$d\langle\hat{s}\rangle/d\tau = 2\vec{\omega}\times\langle\hat{s}\rangle \tag{4}$$

with the initial conditions $\langle\hat{s}(0)\rangle = \{0;0;1\}$ and $P_{\nu_\mu\to\nu_e} = (1 - \langle\hat{s}_z\rangle)/2$. The system of three equations (4), which describes the spin precession in a variable magnetic field $\omega(\tau)$, is equivalent to the system used in Ref. 3.

The results of a numerical solution of Eq. (3) for a realistic model for the density distribution of matter in the Earth[5] is shown in Figs. 1 and 2. Figure 1 is a plot of $P(\nu_\mu\to\nu_e)$ versus E_ν in the case of the motion of neutrinos along the Earth's diameter. We see that as E_ν approaches its "resonant" value corresponding to $l_\nu/l_0^{(c)} = \cos 2\theta_\nu$ (Ref. 3) ($l_0^{(c)}$ corresponds to the density of matter at the Earth's core), the oscillation amplitude increases and at $\Delta m^2/E_\nu = 0.77\times10^{-3}$ eV^2/GeV is approximately sixty times higher than that in a vacuum. P turns out to be extremely sensitive to ρ: a 10% change in ρ changes P by $\sim 50\%$. This behavior sugggests that the "oscillation" method is much more sensitive for measuring ρ than the absorption method,[5] for which $\Delta\mathscr{F}_\nu/\mathscr{F}_\nu \sim 10^{-4} E_\nu$ (GeV) $\Delta M/M$. At 10^{-2} eV$^2 \lesssim \Delta m^2 \lesssim 1$ eV^2 the oscillations can be detected (with the given accuracy of measuring the mass M along the beam path) in a time 10^3–10^7 times shorter than the time it takes to detect them by the absorption method at the same neutrino energies E_ν. It is important to note that, in contrast with the absorption method which is sensitive to only M, in the oscillation method we can, in principle, reconstruct the linear density distribution

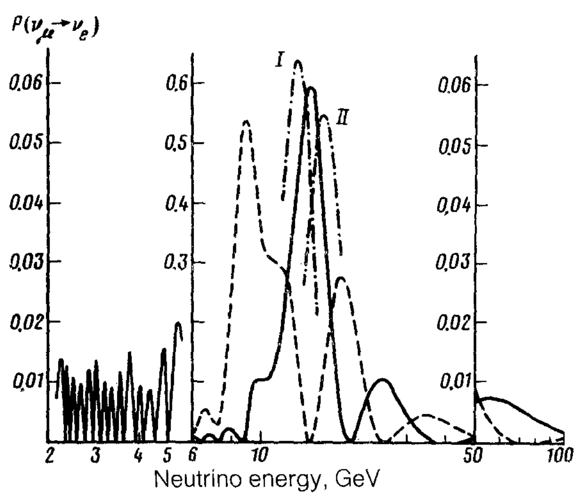

FIG. 1. $P(\nu_\mu\to\nu_e)$ versus E_ν at $x = 12\,800$, $\Delta m^2 = 10^{-2}$ eV^2, and $\sin^2 2\theta_\nu = 10^{-2}$. Solid curve—Model B for the density distribution in the Earth; dashed curve—model A (Ref. 5); dot-dashed curves—$\rho_{\rm I} = 1.1\,\rho_B$ (I); $\rho_{\rm II} = 0.9\,\rho_B$ (II). The scale on the left side of the figure is different from that on the right side.

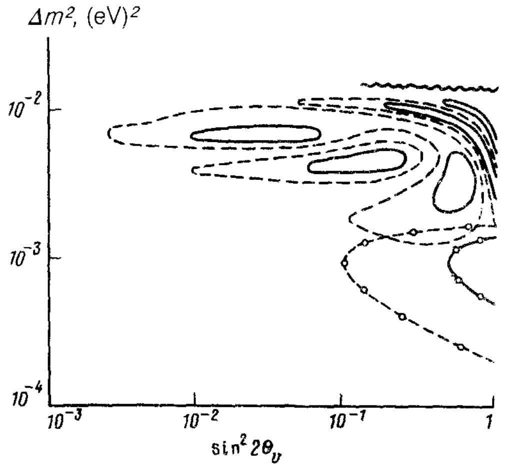

FIG. 2. The lines for the function $P(\nu_\mu \to \nu_e)$ at $x = 12\,800$ km, $E_\nu = 10$ GeV, and $\rho(x) = \rho_B$. Solid curves—$P = 0.5$; dashed curves—$P = 0.1$. O—The lower branches of the corresponding curves for $\rho = 0$.

$\rho(x)$ along the beam by measuring P at different E_ν (the analog of the one-dimensional inverse scattering problem). If $\Delta m^2 \sim 10^{-1}$–10^{-3} eV^2, the maximum neutrino energies needed for such experiments are well within the capabilities of the largest existing accelerators and those of the next generation. The feasibility of such experiments is illustrated by the following examples. The UNK accelerator with $E_p = 3$ TeV, which is now under construction, will produce[6] quasimonochromatic neutrino beams with $\Delta E/E_\nu \cong 5\%$ in the energy range $E_\nu = 20$–250 GeV. At the rate flux density $N_p = 6 \times 10^{14}$ protons per pulse and a length of the decay channel ~ 1.5 km, in the detector with a 20-m radius, separated a distance $L = 6400$ km (α beam $= 30°$), two or three "accompanying" muons (produced in the interaction of the neutrinos in matter in front of the defector) will be detected per pulse or $(4$–$6) \times 10^4$ muons per month of operation. This is a considerably higher value than the minimum required statistical base ($N_\mu \lesssim 100$), since the predicted effect amounts to $P \cong 0.1$–1. In the case of the existing U-70 accelerator ($E_p = 70$ GeV), several months would be required to obtain the necessary statistical data under the same conditions.

The effect under consideration can also increase the sensitivity of the experimental search for the oscillations at $\theta_\upsilon \ll 1$. Figure 2 shows the contour lines for $P(\nu_\mu \to \nu_e)$ at the plane Δm^2, $\sin^2 2\theta_\upsilon$. In the case of a nonmonochromatic beam, the fine structure of the contour lines averages out. However, in the case of a narrow-band spectrum, $\Delta E_\nu / E_\nu \lesssim 0.1$, the curves in Fig. 2 change only slightly. Also shown, for comparison, in Fig. 2 are the lines at the level $\rho = 0$. We see that for $\sin^2 2\theta_\upsilon \cong 1$ the lines for $\rho \neq 0$ lie above those for $\rho = 0$, which corresponds to a decrease in the sensitivity of the oscillation experiments in matter, as pointed out in Ref. 2. At $\sin^2 2\theta_\upsilon \ll 1$, however, there is a region on the plane Δm^2, $\sin^2 2\theta_\upsilon$ where P is equal to unity because of the intensification in matter. This circumstance greatly facilitates the search for oscillations at Δm^2 (eV^2)$/E_\nu$ (GeV) $\sim 10^{-3}$. The values of Δm^2, $\sin^2 2\theta_\upsilon$ and $\rho(x)$ can be determined simultaneously through combined experiments at different E_ν and ρ.

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