Heterotic string in superspace

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A geometric formulation is offered for the theory of a heterotic string in a superspace of dimensionality $(10+496)+16$. The fermion and boson gauge-symmetry laws for the world sheet are found in this superspace.

Heterotic string theory[1] is currently regarded as the leading candidate for the position of a unified theory of the fundamental interactions. At low energies this theory corresponds to an $N=1$, $d=10$ supergravity which is interacting with Yang-Mills fields with the $G=E_8\times E_8$ or SO(32) group.

Although the heterotic string theory has been offered in various formulations,[1,2] no geometric formulation of the theory without anomalies has been constructed. The lack of an understanding of the geometric meaning of heterotic string theory has left even the authors of this theory[1] unsatisfied with it.

Our purpose in the present paper is to construct a geometric formulation of heterotic string theory without anomalies. Our underlying principle is the systematic construction of an action of a heterotic string in a superspace; limitations on the geometry of this superspace will be found from the fermion and boson gauge symmetry of the world sheet. These restrictions turn out to be weaker than the conditions which determine the classical superspace of the theory of Ref. 3. Furthermore (in particular), we find the structures required to eliminate the anomalies from the Green-Schwarz mechanism. The theory is formulated in a superspace with $10+496=506$ boson coordinates (the gauge degrees of freedom are related to the local coordinates on group G; i.e., in our case, there are 496 of them) and with 16 fermion coordinates.

We write the action of the heterotic string in superspace as

$$I=\int d^2\sigma V^{-1}\{\Pi_+^{\mathcal{A}}\Pi_-^{\mathcal{B}}\eta_{\mathcal{A}\mathcal{B}}-1/2\lambda_{++}\Pi_-^{\hat{a}}\Pi_-^{\hat{b}}\eta_{\hat{a}\hat{b}}\}+\int\tilde{T}^A\wedge E_A\,, \tag{1}$$

where σ^+ and σ^- are coordinates on the light cone of the world sheet;

$$\frac{\partial}{\partial\sigma^+}=\frac{1}{\sqrt{2}}\left(\frac{\partial}{\partial\tau}+\frac{\partial}{\partial\sigma}\right),$$

$$\frac{\partial}{\partial\sigma^-}=-\frac{1}{\sqrt{2}}\left(\frac{\partial}{\partial\tau}-\frac{\partial}{\partial\sigma}\right),$$

$V_{\mathbf{a}}{}^{\mathbf{m}}(\sigma^+,\sigma^-)$ is a two-dimensional frame of reference; $\mathbf{a,m}=+,-$; $V=\det(V_{\mathbf{a}}{}^{\mathbf{m}})$; $\Pi_{\pm}{}^A=V_{\pm}{}^{\mathbf{m}}(\partial_{\mathbf{m}}Z^M)E_M{}^A$; $Z^M=(x^m,x^{\hat{m}},\theta^\mu)$; $Z^M=Z^M(\sigma^+,\sigma^-)$, $E_M{}^A(Z)$ is a frame of reference in a space of dimensionality $(10+496)+16$; $A=(a,\hat{a},\alpha),m,a=0,1,\ldots 9$, i.e., the x^m are the coordinates $d=10$, $\hat{m},\hat{a}=1,\ldots,496$ [i.e., the $x^{\hat{m}}$ are the coordinates in the $E_8\times E_8$ group or the SO (32)]; $\mathcal{M}=(m,\hat{m})$, $\mathcal{A}=(a,\hat{a})$; $(\mu,\alpha)=1,\ldots,16$, i.e., θ^μ is the Majorana-Weyl spinor in $d=10$. All the coordinates Z^M are scalars in two dimensions. The Lagrange multiplier λ_{++} is the doubly self-dual part of the traceless symmetric tensor $\lambda_{\mathbf{ab}}$.

The last term in the action, the so-called Wess-Zumino term in the nonlinear chiral model, is defined as a three-dimensional integral. We use the vielbein and twisting forms,

$$E^A=d\sigma^i(\partial_i Z^M)E_M{}^A,\ i=1,2,3,\quad T^A=dE^A+E^B\wedge\Omega^A{}_B.$$

The structure group is defined by

$$\delta_\Lambda E^A=E^B\wedge\Lambda_B^A,\ \delta_\Lambda\Omega^A{}_B=(D\Lambda)^A{}_B, \tag{2}$$

where the connection takes on values in the Lorentz algebra for a, b and α,β and in the G algebra for $\hat{a}$, $\hat{b}$.

The action in (1) is invariant under transformations of the structure group of the superspace, SO(1,9)$\times E_8\times E_8$ [or SO (32)]; i.e., this symmetry is related to the parameters $\Lambda_{[ab]}(Z)$ and $\Lambda_{[\hat{a}\hat{b}]}(Z)=f_{\hat{a}\hat{b}\hat{c}}\Lambda^{\hat{c}}(Z)$. Coordinate-independent transformations of the superspace Z^M are standard transformation of a space with 506 boson coordinates x^m and 16 fermion coordinates θ^μ ($\delta_\xi Z^M=\xi^M(Z)=(\xi^{\mathcal{M}},\xi^\mu)$;

$$\delta_\xi(\partial_i Z^M)=\partial_i Z^N\partial_N\xi^M,\quad \delta_\xi E_M{}^A=-(\partial_M\xi^N)E_N{}^A. \tag{3}$$

In contrast with the usual methods for writing the Wess-Zumino term in a superstring,[2] through the use of a 2-form $B=(1/2!)E^BE^AB_{AB}$ or a 3-form $H=dB+\ldots$, in our formalism it is not necessary to introduce in the theory any new entities beyond vielbeins and a connectedness[1)]: $H=E_A\tilde{T}^A=(1/2)E^AE^BE^CT_{BC,A}$. For the 4-form $d(T^A\wedge E^A)$ we introduce the notation Q.

The symmetry required of the world sheet for the noncontradictory incorporation of the interaction of a heterotic string with a background $E_M{}^A(Z^M),\Omega^A{}_B(Z^M)$, consists of a fermion symmetry gauge with the parameter $k_{+\alpha}(\sigma^+,\sigma^-)$, where $k_{+\alpha}$ is a $d=10$ Majorana-Weyl spinor and a $d=2$ self-dual vector, and a boson gauge symmetry with the parameter $\epsilon(\sigma^+,\sigma^-)$. The action in (1) is invariant under the following k-transformations:

$$\delta_k Z^M = \frac{i}{2} k_{+\alpha} \gamma_a^{\alpha\beta} \Pi_-^a E_\beta^M, \quad \delta_k(V^{-1}\lambda_{++}) = -V^{-1}(\delta_k V_+^{\mathbf{m}}) V_{\mathbf{m}}^-, \tag{4}$$

$$\delta_k (V_+^{\mathbf{m}}) V_{\mathbf{m}}^- = -k_{+\alpha}(\Pi_+^\alpha + T^{\alpha\hat{\beta}}\Pi_{+\hat{\beta}} + T^{\alpha\hat{b}}\lambda_{++}\Pi_{-\hat{b}}). \tag{5}$$

Here are the necessary conditions which must be satisfied if action (1) is to be invariant under (4) and (5) (in addition to the standard Bianchi identities, which relate the twisting and the curvature, the latter taking on values in the algebra of the structure group):

$$T_{\beta\alpha}^{a} = -i\gamma_{\beta\alpha}^{a}, \quad T_{\beta\alpha}^{\hat{a}} = 0, \quad T_{b\alpha}^{\hat{\alpha}} = -i\gamma_{\alpha\beta}^{b} T^{\beta\hat{\alpha}}, \quad T_{\hat{b}\alpha}^{\hat{\alpha}} = 0, \tag{6}$$

$$Q_{ABC\alpha} = 0. \tag{7}$$

All these conditions hold in the superspace corresponding to the classical theory. Here we have[2] $T^{\alpha\beta} = \{(\gamma^b\lambda)^\alpha, \chi^{\hat{b}\alpha}\}$. In the classical theory, however, a condition stronger than (7) holds: Specifically, $Q = 0 \Rightarrow Q_{ABCD} = 0$, which corresponds to[3] $dH = F\wedge F$ in the space (x^m, θ^μ). Condition (7) means that, in particular, we have $Q_{abcd} \neq 0$ and that the latter quantity may contain terms required for eliminating anomalies, $Q_{abcd} = \mathrm{tr}(R_{ab}R_{cd})$.

The local boson symmetry of the world sheet generalizes the transformations of Ref. 5 to a curved space:

$$\delta_\epsilon Z^M = \Pi_-^{\hat{a}} E_{\hat{a}}^M e_{\hat{a}}^M \epsilon, \quad \delta\lambda_{++} = 2\nabla_+\epsilon + \epsilon\overleftrightarrow{\nabla}_-\lambda_{++}. \tag{8}$$

Action (1) is invariant under transformations (8) if the following conditions on the geometry of the space Z^M hold:

$$Q_{ABC\hat{a}} = T_{a}{}^{\hat{a}}{}_{\hat{b}} = T_{\alpha}{}^{\hat{a}}{}_{\hat{b}} = 0, \tag{9}$$

$$T_{\hat{b}}{}^{\hat{a}}{}_{\hat{c}} = -T_{\hat{b}}{}^{\hat{c}}{}_{\hat{a}}. \tag{10}$$

Relations (9) agree with the Bianchi identities, while (10) is satisfied because $T_{\hat{b}}{}^{\hat{a}}{}_{\hat{c}}$ are the structure constants of group G.

In summary, we have constructed an action of a heterotic string in a superspace with 506 boson dimensions and 16 fermion dimensions. This action, which is invariant under fermion k-symmetry and boson ϵ-symmetry of the world sheet, corresponds to a refined theory of supergravity which is interacting with a Yang-Mills theory ("refined" in the sense that there are no anomalies).

[1]In this connection see Ref. 4, where a similar modification of the theory was made for a $d = 11$ supergravity.

[2]These constraints differ from those used in the theory of Ref. 3 (furthermore, in Ref. 3 there is no space

$x^{\hat{m}}$, and there are no derivatives $\mathscr{D}_{\hat{a}}$) in that there is a simultaneous redefinition of the connections and the twistings of the form $\delta\Omega_{\alpha}{}^{a}{}_{b} = (\gamma^{ab}\lambda)_{\alpha}, \delta\Omega_{a}{}^{b}{}_{c} = T_{a}{}^{b}{}_{c}, \ldots, \delta T^{A} = E^{B} \wedge \delta\Omega^{A}{}_{B}$.

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