Influence of relativistic effects on the level widths of baryonium

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Estimates have shown that the relativistic effects change the level widths of baryonium by no more than a factor of 2 or 3 down the energy scale. The earlier predictions, based on the potential scheme, concerning the existence of a spectrum of relatively narrow states (on the order of 10 MeV) in the $N\bar{N}$ system remain in force even when the relativistic corrections are taken into account.

In several papers (see the review in Ref. 1) it was predicted that the $N\bar{N}$ system has a rich spectrum of narrow bound states and resonant states. The problem of the existence of narrow states (relative to the long-lived states) of baryonium has so far remained unresolved. This problem is now being studied extensively.

Although baryonium is a nonrelativistic bound state of $N\bar{N}$, its width is determined by the wave function at short range, $\sim 1/m$, which in turn is related to the value of the wave function at large relative momenta, $q \sim m$. To determine the extent to which the predictions of the nonrelativistic model are reliable, we will determine in this letter the effect of the relativistic effects on the baryonium width.

The nonrelativistic theory gives the following formula for the annihilation width in the $N\bar{N}$ system[1]:

$$\Gamma = v\bar{\sigma} \, | \, \psi(0) \, |^2 \, , \tag{1}$$

where

$$v\bar{\sigma} \approx \pi / m^2 \, , \qquad \psi(0) = \int \tilde{\psi}(\mathbf{p}) \frac{d^3 p}{(2\pi)^3} \quad . \tag{2}$$

Here ψ_0 and $\tilde{\psi}(\mathbf{p})$ are the wave functions of baryonium in the coordinate space (for $r = 0$) and the momentum space, respectively.

The relativistic effects change expression (1), which expresses the width Γ of baryonium in terms of the wave function, and change the wave function itself at $q \sim m$.

The expression for Γ, with allowance for the relativistic effects, can be found by calculating the amplitude of the annihilation diagram in Fig. 1. In this diagram the $N\bar{N}$ state is described by the wave function at the light front. A corresponding method of wave functions on the invariant surface of the light front was developed in Ref. 2. The diagram in Fig. 1 corresponds to a three-dimensional diagram technique which was proposed in Ref. 3 and formulated in Ref. 2 for the case of the light front. The dashed line in the diagram corresponds to a fictitious particle—a spurion. A straightforward calculation of the diagram in Fig. 1 shows that $\psi(0)$ in (1) is replaced by an integral:

$$I = \int \tilde{\psi}(\mathbf{q}) \frac{m \, d^3 q}{(2\pi)^3 \, \epsilon(\mathbf{q})} \quad , \tag{3}$$

where $\epsilon(\mathbf{q}) = \sqrt{\mathbf{q}^2 + m^2}$.

The wave function $\psi(\mathbf{q})$ cannot be calculated in the relativistic region, since the potential model can no longer be used, while the core of the relativistic $N\bar{N}$ interaction (as in the case of the NN system) is not known. To estimate the relativistic effects which lead to a change in the parametrization of $\tilde{\psi}(\mathbf{q})$ at $q \sim m$ in comparison with the relativistic case, we will make use of the concept of the relativistic coordinate space[4] which can be found by means of an expansion in functions[5] that give rise to an irreducible unitary infinite-dimensional Lorentz group representation. In the case of the S-wave under consideration, the expansion of the wave function has the form

$$\tilde{\psi}(q) = \frac{4\pi}{q} \int_0^\infty \psi(\rho) \sin(m\eta\rho) \rho \, d\rho \, , \tag{4}$$

where

$$\eta = \ln \frac{\epsilon(q) + q}{m}$$

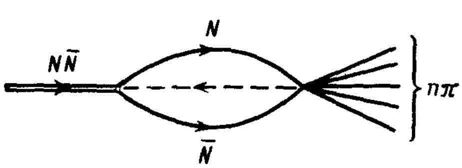

FIG. 1.

is the velocity. At $q \ll m$ Eq. (4) becomes the standard Fourier transformation. The feasibility of such parametrization of the relativistic nuclear wave functions was discussed in Ref. 6. We will use a nonrelativistic wave function for $\psi(\rho)$ in (4). There is no reason to assume that from Eq. (4) we can find the true wave function at relativistic momenta. There is no doubt, however, that Eq. (4) gives the correct order of magnitude of the relativistic effects, since these effects distinguish Eq. (4) from the Fourier transformation. Equation (4) fundamentally changes the behavior of $\tilde{\psi}(q)$ at large q. If the nonrelativistic wave function $\tilde{\psi}(p)$ decreases as p^{-n}, $\tilde{\psi}(q)$ will behave as $q^{-1}[\ln(q/m)]^{-(n-1)}$. Since integral (3) with such a wave function diverges, it should be cut off at the upper limit at $q \sim m$ (which is automatically accomplished by the amplitude on $N\bar{N} \to n\pi$ in the diagram in Fig. 1). Below we will study its sensitivity to the cutoff parameter.

Finally, we can estimate the width heuristically in one more way by substituting $\psi(\rho = 0)$ into (1), where $\psi(\rho)$ can be determined from $\tilde{\psi}(q)$ by transforming Eq. (4):

$$\psi(\rho) = \frac{4\pi m}{\rho} \int_0^\infty \tilde{\psi}(q) \sin(m\eta\rho) \frac{q\,dq}{\epsilon(q)} \qquad (5)$$

As the $\tilde{\psi}(q)$ in (5) we use the nonrelativistic wave function.

The calculations were carried out with use of wave functions corresponding to typical values of the radius and binding energy of the $N\bar{N}$ states.[1] To find an estimate,

TABLE I

Binding energy E and mean radius $\langle r \rangle$	Parameters of the potential	The ratio $\Gamma_{\rm rel}/\Gamma_{\rm nonrel}$ of the width with relativistic corrections to the nonrelativistic width		
		1	2	3
I $E = -100$ MeV $\langle r \rangle = 0.91$ fm	Square well $U_0 = 251.3$ MeV $r_0 = 1.18$ fm	0.9	1.15	0.83
	Hulthén's potential $U_0 = 27.3$ MeV $a = 5$ fm	0.47	0.34	0.38
II $E = -30$ MeV $\langle r \rangle = 1.5$ fm	Square well $U_0 = 89.3$ MeV $r_0 = 1.83$ fm	0.95	1	0.9
	Hulthén's potential $U_0 = 28$ MeV $a = 3$ fm	0.63	0.55	0.53

Column 1—The ratios of the widths calculated from Eq. (1), in which as the $\psi(0)$ we substituted integrals (2) and (3); as $\tilde{\psi}(q)$ and $\tilde{\psi}(p)$ we used the nonrelativistic wave function. Column 2—The same as in 1 but $\tilde{\psi}(q)$ was calculated from Eq. (4) with the nonrelativistic $\psi(\rho)$. Integral (3) is cut off at $q = 1$ GeV/c. At $q = 5$ GeV/c the width increases by a factor of 2. Column 3—The ratio of the width calculated from Eq. (1); $\psi(0)$ was calculated from Eq. (5) for $\rho = 0$ and also from Eq. (2).

we used the wave functions in the square-well potential (U_0 in depth and r_0 in radius) and in Hulthén's potential, $U(r) = -U_0 e^{-r/a}/(1-e^{-r/a})$ with parameters that allow the following values of the binding energy and of the mean radius of baryonium: 1) $E = -100$ MeV, $\langle r \rangle = 0.91$ fm; 2) $E = -30$ MeV, $\langle r \rangle = 1.5$ fm. The widths in this case are on the order of several MeV. The parameters of the potentials and the ratios of the widths, with allowance for the relativistic effects determined by various methods described above, to the width without the relativistic corrections are listed in Table I. We see that the relativistic effects can change the level widths by a factor of nearly 2 or 3. In our calculation, this change occurs down the width scale. The wave function decreases at the origin, which corresponds to a kind of additional repulsion at short distances. The relativistic-effect-induced effective repulsion at short distances was detected by several other methods.[7-10]

In summary, although the relativistic effects in baryonium may be appreciable, they do not alter the predictions[1] concerning the existence of the narrow-state spectrum in the $N\bar{N}$ system.

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