# Multibeam streaming of heavy holes in germanium

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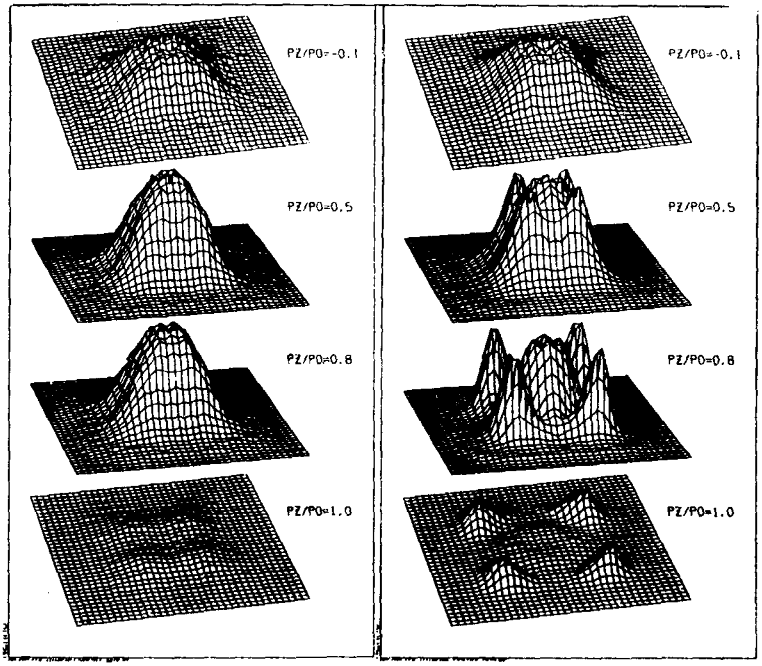
During streaming in an electronic field **E**‖[001], the single-beam distribution of hot holes converts into a multibeam distribution in a magnetic field **H**‖**E**‖[001]. Manifestations of this distribution in the cyclotron-resonance spectrum of germanium hot holes are discussed briefly.

"Streaming" is a distribution of hot charge carriers which is elongated along the direction of the electric field in momentum space in a semiconductor with a strong coupling between carriers and optical phonons.[1] Streaming is involved in several suggestions regarding the attainment of inverted distributions and a negative differential conductivity in semiconductors; the realization of these situations led to the discovery of the induced emission of hot carriers in germanium in the wavelength interval $\lambda \sim 0.07$–4 mm (see Ref. 2, for example). In particular, an induced cyclotron emission of heavy holes with negative masses has been observed[3] and studied[4] during streaming of heavy holes in germanium in fields **E**‖**H**‖[001]. The streaming of heavy holes in germanium and the possibility of an induced cyclotron emission were analyzed in Ref.

5 on the basis of an axisymmetric dispersion of the heavy holes, $\mathscr{E}(\mathbf{p})$. In the present letter we analyze the streaming of holes with the actual dispersion, and we find some new features in the hole distribution in momentum space. These new features are important both for the development of fundamental ideas regarding streaming and for reaching an understanding of the conditions for the appearance of, and the characteristics of, induced emission of holes with negative cyclotron masses.

During streaming, the distribution function is shaped by the collisionless motion of carriers in the electric field at energies $\mathscr{E} < \hbar\omega_0$ up to the energy of an optical phonon, $\mathscr{E} = \hbar\omega_0$, followed by a rapid inelastic scattering at $\mathscr{E} > \hbar\omega_0$, which results in a return of the carriers to energies $\mathscr{E} < \hbar\omega_0$. At $H = 0$ the transverse momentum of a hole ($\mathbf{p}_\perp \perp \mathbf{E}$) is conserved in the motion of holes at energies $\mathscr{E} < \hbar\omega_0$. At $H \neq 0$, the holes execute, in addition to a translational motion, a cyclotron revolution in the plane transverse with respect to the field $\mathbf{H}$. As a result, the distribution function changes, and multibeam streaming arises. The distribution function in fields $\mathbf{E}, \mathbf{H} \| [001]$ has been calculated by the Monte Carlo method. In this numerical simulation, whose



a b

FIG. 1. Distribution function of heavy holes in germanium in various cross sections $p_z = \text{const}$, calculated by the Monte Carlo method for $T = 20$ K, $E = 125$ V/cm, and (a) $H = 0$ or (b) $H = 25$ kOe. The transverse momenta ($p_x, p_y$) vary from $-p_0$ to $p_0$ at a step of 0.05 (the step in the grid); $p_0 = (2\hbar\omega_0\bar{m})^{1/2}$; $\bar{m} = 0.35m_0$.

results are shown in Fig. 1, we used the dispersion

$$\mathscr{E}(\mathbf{p}) = \frac{1}{2m_0}\{Ap^2 - [B^2p^4 + C^2(p_x^2p_y^2 + p_y^2p_z^2 + p_z^2p_x^2)]^{1/2}\},$$

$$A = 13.27, B^2 = 74.48, C^2 = 153.8.$$

We took the scattering by optical and acoustic phonons into account, taking an average of the overlap factor of the hole wave functions over angle.

In the distribution function at $H = 0$ and also at $H \neq 0$, we find a fourfold symmetry with respect to the [001] axis, which reflects the symmetry of the dispersion in this crystallographic direction. The breakup of the streaming into many beams in a strong magnetic field **H** is not difficult to understand by analyzing the dynamics of holes between collision events. In the course of their revolutions and translations, the holes undergo a change in the cyclotron frequency $\omega_c(p)$. If the change in $\omega_c$ over each orbit of a hole is slight, the area ($S$) bounded by the trajectory of the hole in the plane transverse with respect to the field $H$ is conserved in the course of the motion. This area is an adiabatic invariant of the motion. The condition for adiabaticity, $S =$ const, is violated if, in the course of its motion, a hole reaches the surface of the cone $\omega_c(\mathbf{p}) = 0$ (Fig. 2). Consequently, holes that start at $p_z > 0$ inside the cone of negative cyclotron masses ⟨region III in Fig. 2) or inside the cone of positive cyclotron masses (region II) become magnetized and drawn into the revolutionary-translational motion around the axes of the corresponding cones, while retaining a constant area $S$. Holes that start in region I (or, if $p_z > 0$, in regions II and III) pass through the cone $\omega_c = 0$ ($p_z > 0$) when they reach it and either become magnetized in region II or become extended along the electric field into region III and become magnetized there. In this

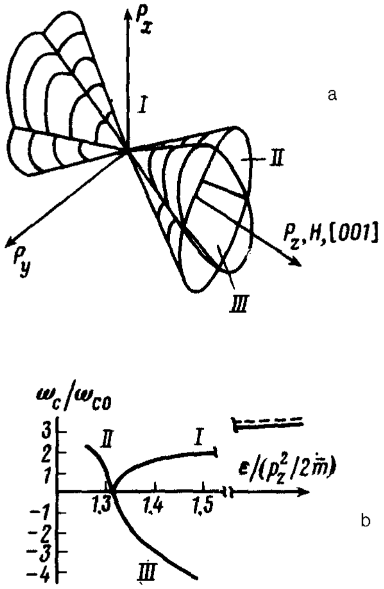


FIG. 2. a—The surface $\omega_c(p) = 0$ in the case $\mathbf{H}\|[001]$ (schematic); b—spectrum of hole cyclotron frequencies, $\omega_c = \omega_c(\mathscr{E}, p_z)$, $p_z = 0.5p$.

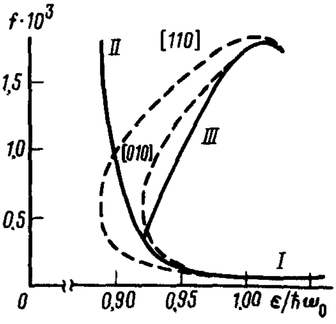


FIG. 3. Solid line—Hole energy distribution for $p_z = 0.8p_0$, $H = 25$ kOe, $E = 125$ V/cm, $T = 20$ K; dashed line—the same, for $H = 0$.

manner, a five-beam streaming arises in fields $\mathbf{H}\|\mathbf{E}\|[001]$: the central beam in the cone of negative cyclotron masses (III) along the [001] axis and four side beams in cones of positive cyclotron masses (II) along directions near [111] axes. The multibeam streaming remains constant over broad ranges of $E$ and $H$; the distribution shown in Fig. 1b is established at $H/E \gtrsim 80$ Oe · cm/V and remains essentially constant as the magnetic field is intensified. In a strong magnetic field, the number of holes with negative effective masses amounts to about 20% of the total number of heavy holes (in the case $H = 0$, 30% of the holes correspond to the same region in phase space). At $T = 77$ K, the relative number of such holes decreases to 8%.

During streaming, the distribution function of the holes with negative masses becomes inverted: $(\partial f/\partial \mathscr{E})p_{z\,=\,\mathrm{const}} > 0$ (Fig. 3). Associated with these inverted holes is the induced cyclotron emission mentioned earlier; this emission has been analyzed elsewhere both in the linear approximation with an axisymmetric dispersion[5] and by numerical simulation.[6,7] The differences in the values of the negative differential conductivity in Ref. 5, on the one hand, and Refs. 6 and 7, on the other, presumably stem from the neglect of the multibeam nature of the streaming in Ref. 5.

The localization of a significant fraction of the holes in the side beams should lead to a line with a mass $m_c \cong 0.4\ m_0$ in the cyclotron-resonance spectrum. This mass would correspond to a quasiharmonic revolution of holes near axes in the regions (II) (a similar feature was observed in the cyclotron-resonance spectrum in crossed fields $\mathbf{E}\perp\mathbf{H}$ in Ref. 8). In an electric field, the relative number of such holes is insignificant, and they make no special contribution to the cyclotron-resonance spectrum.

In summary, the multibeam nature of the streaming of holes in germanium in a magnetic field $\mathbf{H}\|\mathbf{E}\|[001]$ is associated with a manifestation of the particular features of the collisionless motion at $\mathscr{E} < \hbar\omega_0$. Corresponding effects should evidently arise at $\mathbf{H}\|\mathbf{E}[001]$ in other $p$-type semiconductors. Of primary interest here is $p$-Si, where the anisotropy of the dispersion at energies $\Delta/3 < \mathscr{E} < \hbar\omega_0$ ($\Delta$ is the spin-orbit splitting energy) is considerably more pronounced than in germanium.

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