Anisotropy of magnetophonon resonance of short-wave phonons

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The structure of the matrix element and the particular features of momentum transfer by a Landau electron in a quantizing magnetic field lead to an anisotropy of the magnetophonon resonance of a pair of short-wave phonons with a small resultant momentum.

The Hamiltonian of the electron-(two-phonon) interaction in a nonmetallic crystal can be found by singling out the parts of the electron-phonon interaction operator

$$\hat{H}_{e-ph}(\mathbf{r}) = \sum_{\mathbf{q},\mathbf{G}} e\exp(i(\mathbf{q}+\mathbf{G})\mathbf{r})(\epsilon^{-1}\delta\varphi + \tilde{\chi}^{(2)}\delta\varphi\delta\varphi'),$$

which are quadratic in the phonon operators. Here $\delta\varphi$ is the perturbing potential of the ions, $\epsilon^{-1}(\mathbf{q}+\mathbf{G},\mathbf{q}+\mathbf{G}')$ is the reciprocal dielectric matrix of the fluid of valence

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electrons of the semiconductor,[1] and $\tilde{\chi}^{(2)}(\mathbf{q}+\mathbf{G},\mathbf{q}_1+\mathbf{G}',\mathbf{q}_2+\mathbf{G}'')$ is the intrinsic part (analytic in $\mathbf{q}$) of the nonlinear response of the Bloch valence electrons to the field of the perturbed ion potential.[2] The elements of ϵ^{-1} and $\tilde{\chi}^{(2)}$ which are not diagonal in the momentum, must be taken into account, since they stabilize the crystal lattice in the semiconductor. At a small resultant-momentum transfer q, the Hamiltonian contains terms of a polarization type, $\hat{H}^{(2)}_{pol}\sim 1/q$ quadrupole terms $\hat{H}^{(2)}_{pol}\sim q_\mu q_{\mu'}/q^2$ and deformation contributions, which tend toward zero at a small momentum transfer q. For the polarization terms which are most singular in the limit $q\to 0$, the Hamiltonian can be written

$$H^{(2)pol}_{e-ph}(\mathbf{q}) = \sum_{\alpha,\alpha'}\sum_{\mathbf{f}_1 j_1}\sum_{\mathbf{f}_2 j_2}\frac{2\pi e q_\mu Z^{(2)}_{\mu\lambda\lambda'}(\alpha,\alpha')}{\Omega_0\,\epsilon_\infty(\hat{\mathbf{q}})\,q^2}$$

$$\times\frac{\hbar}{2N}\frac{e_\lambda(\alpha|\mathbf{f}_1 j_1)\,e_{\lambda'}(\alpha'|\mathbf{f}_2 j_2)}{(\omega(\mathbf{f}_1 j_1)M_\alpha\omega(\mathbf{f}_2 j_2)M_{\alpha'})^{1/2}}[\hat{b}(\mathbf{f}_1 j_1)\hat{b}^+(\mathbf{f}_2 j_2)+\hat{b}^+_1\hat{b}_2+\hat{b}_1\hat{b}^+_2$$

$$+\hat{b}_1\hat{b}_2]\,\delta(\mathbf{f}_1+\mathbf{f}_2,\mathbf{q}), \qquad (1)$$

where the operators $\hat{b}^+(\mathbf{f}_1 j_1)\equiv\hat{b}^+_1$ and $\hat{b}(\mathbf{f}_1 j_1)\equiv\hat{b}_1$ create and annihilate a phonon with a quasimomentum f_1 of the j_1 branch; M_α is the mass of ion α in a unit cell of volume Ω_0; $e_\lambda(\alpha|\mathbf{f}_1 j_1)\lambda$ is a component of the polarization vector for the ion α of the phonon $(\mathbf{f}_1 j_1)$; $\hat{q}=\mathbf{q}/q$; and $Z^{(2)}_{\mu\lambda\lambda'}(\alpha,\alpha')$ is given by the expression

$$Z^{(2)}_{\mu\lambda\lambda'}(\alpha,\alpha')=\sum_{\mathbf{G}\neq 0,\mathbf{G}}\pi^{(1)}_\mu(0,\hat{\mathbf{G}})\epsilon^{-1}(\hat{\mathbf{G}},\mathbf{G})\left[\frac{v_\alpha(\mathbf{G})}{\Omega_0}G_\lambda G_{\lambda'}e^{-i\mathbf{G}\mathbf{u}^{(0)}_\alpha}\delta_{\alpha\alpha'}\right.$$

$$-\sum_{\mathbf{G}'\mathbf{G}''}\sum_{\mathbf{q}_1\mathbf{q}_2}\tilde{\chi}^{(2)}(\mathbf{G},\mathbf{q}_1+\mathbf{G}',\mathbf{q}_2+\mathbf{G}'')$$

$$\times\frac{v_\alpha(\mathbf{q}_1+\mathbf{G}')}{\Omega_0}\rho^{(0)}_\alpha(\mathbf{q}_1+\mathbf{G}')\frac{v_{\alpha'}(\mathbf{q}_2+\mathbf{G}'')}{\Omega_0}\rho^{(0)}_{\alpha'}(\mathbf{q}_2+\mathbf{G}'')$$

$$\left.\times(\mathbf{q}_1+\mathbf{G}')_\lambda(\mathbf{q}_2+\mathbf{G}'')_{\lambda'}\right]. \qquad (2)$$

Here $\pi(\mathbf{q}+\mathbf{G},\mathbf{q}+\mathbf{G}')$ is the polarization operator of the Bloch valence electrons, which, like the inverse dielectric matrix ϵ^{-1}, is nondiagonal in the quasimomentum transfer $\mathbf{q}+\mathbf{G},\pi^{(1)}_\mu(0,\mathbf{G}')=\partial\pi(\mathbf{q},\mathbf{q}+\mathbf{G}')/\partial q_\mu|_{\mathbf{q}=0}$, $v_\alpha(\mathbf{q}+\mathbf{G})$ is the Fourier transform of the pseudopotential of the ions of species α in the crystal, and $\rho^{(0)}_\alpha(\mathbf{q}+\mathbf{G})$ is the partial structure factor of the ideal (unperturbed) crystal. We switch from a summation over the eight atoms in the unit cell to a summation over the two atoms in the primitive cell. We see from (2) that the tensor $Z^{(2)}_{\mu\lambda\lambda'}(\alpha,\alpha')$, which determines the electron-(two-phonon) interaction, consists of two parts. The first part, which corresponds to the first term [below, $\tilde{Z}^{(2)}_{\mu\lambda\lambda'}\delta_{\alpha\alpha'}(-1)^\alpha$] in square brackets in (2), has the symmetry of the ideal crystal. The second term in (2) corresponds to the response to

the $\mathbf{q}_2 + \mathbf{G}''$ mode of a crystal distorted by the $\mathbf{q}_1 + \mathbf{G}'$ mode, so its symmetry is lower than that of an ideal crystal. This term comes into play when the first term vanishes because of the lattice symmetry, e.g., in a semiconductor with the diamond structure. Levinson and Rashba[3] have pointed out the existence of this mechanism for an electron-(two-phonon) polarization interaction. In a semiconductor with the zinc blende structure, on the other hand, the nonlinear polarization interaction is substantially weaker than the linear polarization interaction. In a semiconductor with the zinc blende structure, with a symmetry corresponding to class $43m$, the third-rank pseudotensor corresponding to all the invariance requirements (Ref. 4, for example) accordingly becomes

$$Z^{(2)}_{\mu\lambda\lambda'} = \sum_{\hat{\mathbf{G}} \neq 0, \mathbf{G}} \pi^{(1)}_{\mu}(0, \hat{\mathbf{G}})\, \epsilon^{-1}(\hat{\mathbf{G}}, \mathbf{G}) \frac{v_{\alpha}(\mathbf{G})}{\Omega_0} G_{\lambda} G_{\lambda'} \exp(-i\mathbf{G}\mathbf{u}^{(0)}_{\alpha})$$

$$= \tilde{Z}^{(2)}_{123} \epsilon_{\mu\lambda\lambda'} (-1)^{\alpha}. \qquad (3)$$

Here $\epsilon_{\mu\lambda\lambda'}$ is the completely symmetric third-rank tensor, which vanishes when any two of its indices are the same and which does not change sign upon the permutation of any two indices ($\epsilon_{xyz} = \epsilon_{123} = 1$). The same symmetry requirements lead to a vanishing of the pseudotensor $Z^{(2)}_{\mu\lambda\lambda'}$ in a semiconductor with the diamond structure. In the case of the interaction of an electron with a pair of short-wave phonons with a small resultant momentum, with $\mathbf{f}_1 \approx -\mathbf{f}_2$, the numerator $\mathbf{q}\hat{Z}\mathbf{e}_1\mathbf{e}_2/\hat{M}$ in (1) reduces to

$$\mathbf{q}\hat{Z}\mathbf{e}_1\mathbf{e}_2/\hat{M} = \tilde{Z}^{(2)}_{123} \left\{ q_x \sum_{\alpha=1}^{2} (-1)^{\alpha} e_y(\alpha \mid \mathbf{f}_1 j_1) e^{*}_z(\alpha \mid \mathbf{f}_1 j_2)/M_{\alpha} + \text{permut.}_{x,y,z} \right\}. \qquad (4)$$

The symmetry of the crystal thus imposes restrictions on the directions of the polarization vectors in the pair of phonons emitted during the loss of a momentum $\mathbf{q}$ by the electron. In particular, it follows from (1) and (4) that the emission (or absorption) of two short-wave transverse phonons by an electron occurs along $\mathbf{q}$ by virtue of momentum conservation, $\delta(\mathbf{f}_1 + \mathbf{f}_2, \mathbf{q})$, and the small value of $\mathbf{q}$; in other words, $\mathbf{f}_1$ and $\mathbf{f}_2$ are directed opposite each other and collinear with $\mathbf{q}$. In the absence of a magnetic field, it would be difficult to experimentally observe an anisotropy of the electron-(two-phonon) interaction because of the complexities in distinguishing the contribution of two-phonon processes from the background of single-phonon processes. The application of a quantizing magnetic field to the semiconductor presents an opportunity to single out the processes involving the interaction of a Landau electron simultaneously with a pair of phonons, in particular, in the magneto-two-phonon resonance,[5,6] which consists of oscillations of the transverse conductivity of the semiconductor in a quantizing magnetic field when plotted against the strength of the magnetic field and which has a period determined by the condition

$$\hbar\omega_{ph1}(\mathbf{q}) \pm \hbar\omega_{ph2}(-\mathbf{q}) = e\hbar H / m^{*}c. \qquad (5)$$

It is assumed that a pair of phonons 1 and 2 with a small resultant momentum corresponds to a peak in the sum- or difference-two-phonon state density. In an electric field crossed with a quantizing magnetic field, the transfer of momentum by a

Landau electron occurs, on the average, only in the direction perpendicular to the electric and magnetic fields (the y direction). The transfer of the wave vector across the magnetic field is limited by the inverse magnetic length $q_\perp < 1/\lambda_H$, where $\lambda_H = (c\hbar/eH)^{1/2}$; the longitudinal momentum transfer of a Landau electron, $\hbar q_z$, on the other hand, is determined by energy and momentum conservation in the event in which the pair of phonons is produced under resonance conditions. At liquid-hydrogen temperatures, this transfer is considerably smaller than the reciprocal magnetic length: $q_z < 1/(\lambda_H (2\hbar\omega_H/kT)^{1/2})$, and $\hbar\omega_H \gg kT$. The electron-phonon polarization interaction is most effective at a small momentum transfer $\mathbf{q}$. Nevertheless, nearly the limiting momentum $q_y \sim 1/\lambda_H$ is transferred in scattering events because the Kubo formula for the correlation function of the velocities of the centers of the electron orbits contains a factor q_y^2, which "kills" small transfers of the transverse momentum $\hbar q_y$. We thus have $q_{y\text{eff}} \gtrsim q_{x\text{eff}} \gg q_{z\text{eff}}$. The predominance of q_y also leads to a resultant drift of electrons in the direction perpendicular to $\mathbf{E}$ and $\mathbf{H}$. Nevertheless, in order of magnitude we have $q_{y\text{eff}} \sim q_{x\text{eff}}$, and as a result, we can only assert that the transfer of momentum by a Landau electron in a quantizing magnetic field in an individual event occurs in the plane perpendicular to $\mathbf{H}$, leading to an anisotropy of the magneto-phonon resonance of short-wave phonons (in particular, acoustic phonons). In the interaction of a Landau electron with a pair of short-wave transverse acoustic or optical phonons with a small resultant momentum, the quasimomentum of each of the phonons lies in the plane perpendicular to the quantizing magnetic field. A change in the direction of the quantizing magnetic field in the crystal leads to a change in the direction of the momentum ($\mathbf{q}$) transferred to the pair of phonons and thus, according to (1) and (2), to a change in the direction of the quasimomenta of the phonon pair, $\mathbf{f}_1$, $\mathbf{f}_2$. If a part of the Brillouin zone with a peak in the two-phonon state density lies along the direction of the quasimomentum of one of the phonons, the transverse conductivity of the semiconductor will exhibit magneto-two-phonon oscillations in a plot against H. These oscillations die out upon a subsequent change in the direction of the quantizing magnetic field. At low temperatures, a quantizing magnetic field makes it possible to effectively study the Brillouin zone by means of resonant two-phonon processes (including processes that occur in regions adjacent to the boundaries of the Brillouin zone). The Brillouin zone is scanned along planes perpendicular to the quantizing magnetic field and passing through the center of the zone. Finally, we note that the anisotropy should manifest itself in all magnetooscillation effects with an inelastic scattering by a pair of short-wave phonons: oscillations of the photocurrent in a quantizing magnetic field, a cyclotron-two-phonon resonance, etc.

[1]N. Wizer, Phys. Rev. **129**, 62 (1963).
[2]L. J. Sham, Phys. Rev. **150**, 720 (1966).
[3]I. B. Levinson and É. I. Rashba, Pis'ma Zh. Eksp. Teor. Fiz. **20**, 63 (1974) [JETP Lett. **20**, 27 (1974)].
[4]W. A. Wooster, Tensors and Group Theory for the Physical Properties of Crystals, Oxford Univ. Press, London, 1973.
[5]L. Eaves, R. A. Stradling, S. Askenazy, *et al.*, J. Phys. C. **7**, 1999 (1974).
[6]E. A. Mazur and V. N. Sobakin, Fiz. Tverd. Tela (Leningrad) **18**, 3481 (1976) [Sov. Phys. Solid State **18**, 2026 (1976)].

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