Emission by an accelerated electron

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(Submitted 25 March 1986)

Pis'ma Zh. Eksp. Teor. Fiz. **43**, No. 9, 407–409 (10 May 1986)

This letter analyzes the interaction of accelerated particles (electrons) with vacuum fluctuations from the standpoint of the noninertial frames of reference in which these particles are at rest. This approach leads to a clear picture of certain many-photon processes and an estimate of their intensities.

We know that an observer moving in a line at a constant acceleration α preceives a temperature

$$T = a/2\pi \tag{1}$$

(we are using a system of units with $\hbar = c = k = l$, where k is the Boltzmann constant) as a result of the exertion on this observer of some force of nongravitational origin. This effect, discovered by Davies[1] and Unruh[2] (see also Ref. 3), has been explained on the basis that the metric (the so-called Rindler metric) has a horizon in the rest frame of the observer. After an expansion in the eigenstates of the field in the Rindler metric, the zero-point field vibrations in the Minkowski metric turn out to correspond to this temperature. In this manner, a relationship is established between the temperature sensed by the accelerated observer, on the one hand, and the temperature and radiation of a black hole which were discovered by Hawking,[4] on the other: The black hole also has a horizon at the gravitational radius. A point of fundamental importance is that the radiation perceived by an observer at rest with respect to the Rindler frame of reference is itself accelerated (in the terminology of Unruh and Wald[5]). In other words, an isolated observer sees around himself not a homogeneous thermal bath with the temperature in (1) but a thermal bath which is at equilibrium (in complete accordance with Einstein's equivalence principle) in a constant gravitational field with the metric coefficient $g_{00} = r^2 = x^2 - t^2$. The temperature at the radiation depends on the coordinates in accordance with the familiar formula $T\sqrt{g_{00}}$ (Ref. 6, for example), which relates the temperature at different points of a system at thermal equilibrium with the gravitational potential at these points. This result is in agreement with the proposition that the acceleration of an observer who is at rest with respect to the Rindler coordinates depends on his spatial coordinate: $\alpha = r^{-1}$.

If a system which has several energy levels is in a constant-acceleration motion, it will, after a certain time, be distributed in excited states in accordance with a Boltzmann law,[2] with the temperature of the system being given by expression (1). (Bell and Leinaas[7] have shown, however, that for an electron which is moving along a straight line in a constant electric field the time required to reach thermal equilibrium is fantastically long for the existing linear accelerators.) In a study of the particular features of this distribution, it is important to recall that properties of the accelerated thermal radiation differ from those of radiation which is at a constant temperature. In particular, in a calculation of the state density we need to use the exact wave functions in the Rindler metric: the WKBJ approximation is not sufficient for them in several cases because the thermal wavelength $\lambda = T^{-1}$ is on the order of the scale length of the inhomogeneity. When we take this circumstance into account, we find that the spectrum of vacuum excitations in the Rindler metric, which was calculated by Takagi,[8] is a thermal spectrum, and the anisotropic effects discussed in Refs. 9 and 10 are consistent with the thermal nature of the radiation (see also Refs. 11 and 12).

There is an interesting point to note in this connection. If we calculate the correlation function $\langle \varphi_{,\mu}(x)\varphi_{,\nu}(x')\rangle$, where φ is a massless scalar field, and $\mu,\nu = 1,2,3$, along the trajectory of an observer in uniform acceleration, we find that this function is anisotropic. In the Rindler coordinates, τ,r,y,z we find

$$\langle \partial_y\varphi(\tau,r,y,z)\partial_y\varphi(\tau+\Delta\tau,r,y,z)\rangle = \langle \partial_z\varphi(\tau,r,y,z)\partial_z\varphi(\tau+\Delta\tau,r,y,z)\rangle$$

$$= 1/8\pi^2 r^4(\mathrm{ch}\,\Delta\tau - 1)^2; \qquad (2)$$

$$\langle \partial_r\varphi(\tau,r,y,z)\partial_r\varphi(\tau+\Delta\tau,r,y,z)\rangle = (2-\cosh\Delta\tau)/8\pi^2 r^4(\cosh\Delta\tau - 1)^2;$$

$$r = \sqrt{x^2 - t^2}, \quad \tau = \mathrm{arc\,sinh}\,(t/r).$$

However, we know that the energy-momentum tensor of a scalar field in a plane space-time should instead be written in the following "improved" form[13,14]:

$$T_{ik} = \partial_i\varphi\partial_k\varphi - \frac{1}{2}g_{ik}\partial_m\varphi\partial^m\varphi + \frac{1}{6}(g_{ik}\partial^m\partial_m - \partial_i\partial_k)\varphi^2; \quad T_i^i = 0. \qquad (3)$$

The correlation function corresponding to this T_{ik} is (we are omitting a part $\propto g_{ik}$)

$$\langle \partial_i\varphi(\tau,r,y,z)\partial_k\varphi(\tau+\Delta\tau,r,y,z)\rangle - \frac{1}{6}\frac{\partial^2}{\partial x^i\partial x^k}\langle\varphi(\tau,r,y,z)\varphi(\tau+\Delta\tau,r,y,z)\rangle. \qquad (4)$$

It is easy to see that this function is spatially isotropic, telling us again that the accelerated radiation is locally isotropic.

For a particle revolving uniformly along a circle, there is no Davies-Unruh effect. A quantization of the fields in a uniformly rotating frame of reference after replacement of the photon energy E by the energy in the rotating frame, $E' = E - M\Omega$, where M is the angular momentum of the photon, is no different from quantization in a Minkowski metric.[15] We thus see that the condition $\alpha \neq 0$ is not sufficient for a nonequivalence between the vacuum in the accelerated frame of reference and the vacuum in the Minkowski metric—the condition $dE/ds \neq 0$ is also necessary. An electron revolving along a circle, however, senses vacuum fluctuations, nevertheless. These fluctuations are different from the vacuum fluctuations in the Minkowski metric,[15] which may, in general, lead to a self-excitation of the electron. This excitation is manifested in the known fact that an electron moving through a constant magnetic field has an incomplete radiative polarization.[16] The representation of vacuum fluctuations differing from those in the Minkowski metric is therefore considerably more general than the Davies-Unruh temperature in (1), which can be introduced in only exceptional cases. Nevertheless, the spectrum of vacuum fluctuations perceived by an observer in uniform circular motion at a velocity $\beta \approx 1$ is close to a thermal spectrum with a temperature on the order of (1) (Ref. 15, for example).

We also wish to point out another phenomenon which is associated with the temperature perceived by an accelerated electron: an additional emission by the electron. In a thermal-radiation field, there is a Compton scattering of photons. From the standpoint of the quantum electrodynamics of Minkowski space, this process corresponds to a Feynman diagram which describes the two-photon emission of an accelerated electron (i.e., a diagram of higher order in the fine-structure constant α than the diagram describing multipole single-photon emission), since in this case both the absorption and the emission of Rindler photons correspond to the emission of Minkowski photons.

Before the scattering by the electron, the thermal photons in the Rindler metric are not observable by a fixed (not accelerated) observer, but after the scattering, they become real and observable. We can construct an expression which might yield a crude estimate of the intensity of this emission. Using the photon energy density $\epsilon \approx T^4 \approx a^4$ and the Thomson cross section $\sigma_0 \approx 10\alpha^2 m^{-2}$, where m is the mass of the electron, we conclude that the radiation intensity is $I \approx 10\alpha^2 m^{-2} a^4$. When we transform to a frame of reference in which the electron is moving at a velocity β, the radiation becomes concentrated in an angle $\varphi \approx 1/\gamma$, where $\gamma = 1/\sqrt{1-\beta^2}$. The intensity of the radiation increases by a factor of γ. It is interesting to compare this radiation with the classical radiation by an accelerated electron in its rest frame. Since the intensity of the classical radiation is described by $I_0 = (2/3)\alpha a^2$, we conclude that $I < \alpha I_0$ if $a < m$. For an electron revolving in a magnetic field B, an acceleration $a \approx m$ is reached at $\gamma B \approx m^2/e \approx 5 \times 10^{13}$ G. With $B = 2 \times 10^5$ G, e.g., we would need an electron with an energy of 10^5 GeV. At $a > m$ we need to take into account the fact that the cross section for the scattering of a photon by an electron decreases according to the Klein-Nishina-Tamm formula ($\sigma \approx \sigma_0 m/a$), and the typical energy of the photon after the scattering is $\omega \approx m$. As a result, we find $I/I_0 \approx \alpha$ at $a > m$. This result agrees with the behavior of the other radiative corrections in quantum electrodynamics under the conditions $E, B \gg E_0 = m^2/e\hbar$.

At temperatures $T \gtrsim m$ we see another effect: An accelerated electron must perceive thermal electrons and positrons, since the wave functions of the filled Dirac sea in the Minkowski frame also lead to free electrons and positrons after a transformation to Rindler coordinates. The annihilation of an accelerated electron with such a positron gives rise to photons which are received by a detector at rest. From the standpoint of the quantum electrodynamics of Minkowski space, this process again corresponds to the Feynman diagram for a two-photon emission.

We wish to reiterate that we are not claiming that the expressions describing the two-photon processes here are exact. We simply wish to point out the form which these processes assume in a Rindler frame of reference and some crude but simple methods for describing them quantitatively which follow from the form they take.

L. P. Grishchuk participated in some useful discussions.

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Translated by Dave Parsons