Low-temperature spatial fluctuations of the current in disordered conductors

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At zero temperature in a uniform external field, the spatial fluctuations of the current in a conductor are proportional to the size of the sample. At a nonzero temperature, this size gives way to the coherence length of a normal metal. These fluctuations are accompanied by fluctuations of the chemical potential, which are also proportional to these lengths.

In this letter we report a study of the spatial fluctuations of the current density in disordered conductors at low temperatures, where the conductivity is determined by elastic scattering by impurities. In a particular realization of a random potential over distances less than L_φ (L_φ is the phase relaxation length of the electron wave function, determined by energy relaxation, for example), the current flow is described by the exact solution of a Schrödinger equation. The Schrödinger equation, however, like the Liouville equation, conserves the phase volume and therefore the entropy. The concepts of a conductivity and a mean free path arise after an average is taken over the realizations of the random potential. As an electron undergoes a complicated motion through a random potential, there is a mixing of states in phase space, which is apparently equivalent to an averaging over random realizations.[1] In the present letter we show that spatial fluctuations of the current density at a given point $\mathbf{J}(\mathbf{r})$ arise in a macroscopically uniform conductor in an external electric field $\mathbf{E}(\mathbf{r})$. These fluctuations apparently reflect this mixing; at $T=0$ we have $\langle \mathbf{J}^2(\mathbf{r}) \gg \langle \mathbf{J}(\mathbf{r})\rangle^2$.

Fluctuations of the conductance G (which is the reciprocal of the total resistance of the sample) were studied in Refs. 2–4; it was shown there, in particular, that at $T=0$ we have $T=0|\langle \delta G^2\rangle \cong e^4/\hbar^2$, and these quantities are independent of the dimensions of the sample. We believe that this result is again a consequence of a mixing effect. Here the angle brackets denote an average over realizations of the random potential. Calculations show that in a uniform electric field in the three-dimensional case we have

$$\langle \mathbf{J}^2(\mathbf{r})\rangle = \frac{9\zeta(3/2)}{(2\pi)^{9/2}} \frac{e^4}{\hbar^2} \frac{L_T}{l} \left(\frac{p_F}{\hbar}\right)^2 E^2, \tag{1}$$

$$\langle \mathbf{J}(\mathbf{r})\mathbf{J}(\mathbf{r}')\rangle = \frac{9\zeta(3/2)}{(2\pi)^{9/2}} \frac{e^4}{\hbar^2} \frac{L_T}{l} \frac{E^2}{|\mathbf{r}-\mathbf{r}'|^2} e^{-\frac{|\mathbf{r}-\mathbf{r}'|}{e}}; \quad |\mathbf{r}-\mathbf{r}'| > l \ . \tag{2}$$

In a film or wire of thickness $\alpha < L_T$, additional factors

$$\frac{L_T}{a} \ln \frac{L_\varphi}{L_T} \text{ and } \frac{L_T L_\varphi}{a^2},$$

appear in the corresponding expressions. Here $L_T = \sqrt{D\hbar/T}$, $D = (V_F l)/3$, l is the mean free path of the electrons with respect to scattering by impurities, and p_F is the Fermi momentum. At $T = 0$ we would replace L_T by the sample size L in expressions (1) and (2). It can be seen from (1) and (2) that in the three-dimensional case, under the condition $L_T \gg l(p_F l)/\hbar^2$, we would have $\langle \mathbf{J}^2 \rangle \gg \langle \mathbf{J} \rangle^2 = \sigma_0^2 E^2$; i.e., only a small fraction of the current would flow along the field (σ_0 is the macroscopic conductivity of the sample). The direction of the fluctuational current is not correlated with the direction of the field **E**. The nature of this phenomenon is analogous to an effect discovered in Ref. 5: the absence of correlations in the fluctuations of the components of the conductance tensor $G_{\alpha\beta}$. The presence of these currents, random in magnitude and direction, and stemming from a mixing effect, gives rise to fluctuations of the magnetic-moment density **m**, so that we have

$$\langle \mathbf{m}^2 \rangle \cong \left(\frac{e^2}{\hbar c}\right)^2 E^2 \frac{L_T}{L} . \tag{3}$$

This effect is closely related to a lowering of the symmetry in mesoscopic samples[5] and to the appearance of a magnetization in a low-symmetry crystal on which an electric field is imposed.[6]

We can use an impurity diagram technique to calculate the correlation functions for the currents in (1) and (2). The diagrams of importance here are shown in Fig. 1 (Refs. 3 and 4). The currents in (1) and (2) are dominated by the diagrams of the type in Fig. 1a. The current correlation functions can be related to $E(r)$ by the relation

$$\langle J_\alpha(\mathbf{r}) J_\beta(\mathbf{r}') \rangle = \int d\mathbf{r}_1 d\mathbf{r}_1' K_{\alpha\gamma\beta\delta}(\mathbf{r}; \mathbf{r}_1; \mathbf{r}'; \mathbf{r}_1') E_\gamma(\mathbf{r}_1) E_\delta(\mathbf{r}_1')$$

$$= \int d\mathbf{r}_1 d\mathbf{r}_1' \langle \sigma_{\alpha\gamma}(\mathbf{r}, \mathbf{r}_1) \sigma_{\beta\delta}(\mathbf{r}', \mathbf{r}_1') \rangle E_\gamma(\mathbf{r}_1) E_\delta(\mathbf{r}_1') .$$

As was shown in Ref. 7, the diagrams in Fig. 1a lead to

$$K^{(0)}_{\alpha\gamma\beta\gamma} \sim \frac{\sigma_0^2}{|\mathbf{r}-\mathbf{r}_1|^2 l^4} \times \exp\left\{-\frac{|\mathbf{r}-\mathbf{r}'|}{l} - \frac{|\mathbf{r}_1-\mathbf{r}_1'|}{l}\right\}\delta_{\alpha\beta} \begin{cases} 1 & \text{for } |\mathbf{r}-\mathbf{r}_1| < L_T \\ \dfrac{L_T^2}{|\mathbf{r}-\mathbf{r}_1|^2} & \text{for } |\mathbf{r}-\mathbf{r}_1| > L_T \end{cases} . \tag{4}$$

On the other hand, we have

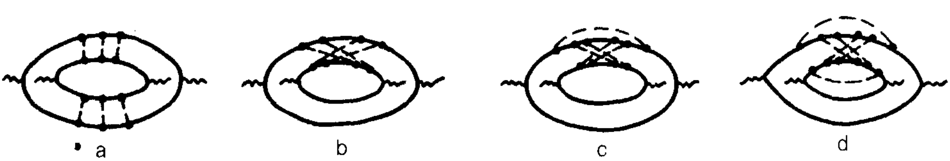

FIG. 1.

$$\langle \sigma(\mathbf{r},\mathbf{r}_1)\rangle \cong \frac{\sigma_0}{|\mathbf{r}-\mathbf{r}_1|^2 l}\, e^{-\frac{|\mathbf{r}-\mathbf{r}_1|}{l}} . \tag{5}$$

This result means that the fluctuational current which arises at the point r under the influence of the electric field at point r_1 falls off far more slowly than does the average current, with the result that we have (1) and (2). The large factor L_T/l arises from fields concentrated in a large region L_T^3.

In the two-dimensional case, under the condition $a \ll |\mathbf{r}-\mathbf{r}_1| \ll L_T \ll L_\varphi$, we have

$$K^{(0)}_{\alpha\gamma\beta\gamma}(\mathbf{r};\mathbf{r}_1;\mathbf{r};\mathbf{r}_1) \sim \delta_{\alpha\beta}\, \frac{\sigma_0^2}{a^2 l^4}\, \ln^2 \frac{|\mathbf{r}-\mathbf{r}_1|}{L_T} . \tag{6}$$

The sum of diagrams b and c in Fig. 1 is zero.[4] The diagrams in part d and also diagrams containing a cooperon, while making a small contribution to (1) and (2), make a contribution of the same order as a to the fluctuations of the conductance, $\langle \delta G^2 \rangle$. However, they describe a correlation between the local (on the scale of l) responses over large distances and at $|\mathbf{r}-\mathbf{r}'| < L_T$ they lead to $K^{(1)}(\mathbf{r};\mathbf{r}_1;\mathbf{r}';\mathbf{r}_1') \sim K^{(0)}(\mathbf{r};\mathbf{r}';\mathbf{r}_1;\mathbf{r}_1')$ ($K = K^{(0)} + K^{(1)}$, in contrast with the classical fluctuational currents, which are correlated at short range. Consequently, they make a contribution to $\langle \delta G^2 \rangle$ which is of the same order of magnitude as that from the diagrams of the type in Fig. 1a.

Experimentally, one can measure current fluctuations by an ordinary four-probe arrangement, with the current passed along the sample while the fluctuational voltage is measured at point contacts. As in the experiments of Ref. 8, it is necessary to study the dependence of the voltage across the contacts on the magnetic field.

As was suggested in Ref. 4, a change in a magnetic field is equivalent to a change in the realization of a random potential. The typical strength of the fields which result in an effective change in realizations is[2] $H \simeq \hbar c/eL_T^2$.

In inversion layers one can study fluctuations of the current upon a change in the gate voltage.[9] Finally, in metallic spin glasses the fluctuations of the currents through a point contact which arise upon a change in the magnetic field are related to spin flipping.[10]

The entire discussion above holds in the case in which the electrons interact with the external electric field but not with each other. Under this condition, continuity of the current is ensured by the appearance of inhomogeneous density fluctuations $\delta\rho(\mathbf{r})$. As H is varied, fluctuations of the chemical potential,

$$\langle \delta\mu^2(\mathbf{r}) \rangle \simeq \left(e\mathbf{E} \frac{\hbar}{pF} \right)^2 \frac{L_T}{l},$$

arise; these fluctuations determine the order of magnitude of the fluctuations of the electric potential in a system of interacting electrons. By virtue of the quantum-mechanical nature of large current fluctuations, these fluctuations are not related to a local production of entropy; they are evidence that $\sigma(\mathbf{r},\mathbf{r}')$ is not self-averaging. Large fluctuations in $\mathbf{E}(\mathbf{r})$ have been observed previously in a study of the screening of an external field in the insulating phase of a doped semiconductor.[11] It was mentioned in that paper that the exponential decay of $\mathbf{E}(\mathbf{r})$ was due exclusively to an averaging of the field over realizations.

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