Mesoscopic behavior of the temperature dependence of the transverse hopping conductivity of an amorphous film

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The transverse hopping conductivity of an amorphous film of small area is shown to experience random oscillations upon a change in temperature.

At reasonably low temperatures the conductivity of amorphous semiconductors is determined by tunneling electron jumps along the impurities whose levels are situated near the Fermi level. The temperature dependence of the conductivity is described by the Mott equation,

$$\sigma = \sigma_0 e^{-(T_0/T)^{1/4}}, \tag{1}$$

where

$$T_0 = \beta' / g a^3 . \tag{2}$$

Here g is the density of the impurity states at the Fermi level, a is the localization length of the electron at the impurity, and $\beta \simeq 20$ is a numerical coefficient.[1] Amorphous semiconductors are usually studied in the form of thin films. Pollak and Hauser[2] were first to advance the idea that the transverse conductivity of a film of sufficiently small thickness L is determined by electron jumps along the widely spaced

chains of impurities which link the opposite surfaces of the film. The distance between the adjacent sites in the chain is considerably shorter than $(a/Tg)^{1/4}$—the typical length of a jump in a bulk material. If the chain has N sites, its conductivity will be

$$\sigma(N, \epsilon) = \sigma_0 e^{-u(N, \epsilon)}, \tag{3}$$

where

$$u(N, \epsilon) = \frac{2L}{Na} + \frac{\epsilon}{T}, \tag{4}$$

$$\epsilon = {}^1/_2 \max_i \{|\epsilon_i| + |\epsilon_{i+1}| + |\epsilon_{i+1} - \epsilon_i|\}. \tag{5}$$

The energy of the i-th site, ϵ_i, is reckoned from the Fermi level. The dominant contribution to the conductivity of the film comes from the optimum chains, i.e., from the chains for which the product of their conductivity $\exp[-u(N,\epsilon)]$ and formation probability $\exp[-\Omega(N,\epsilon)]$ is maximum. The dependence $\Omega(N,\epsilon)$ is given by

$$\Omega = N \ln \frac{\epsilon_0}{\epsilon}, \tag{6}$$

where $\epsilon_0 = 1/gv_0$, and $v_0 = (La)^{3/2}$ is the volume into which an impurity center must find its way in order to become a link in the chain. The minimum of the sum $\Omega + u$ is reached when

$$N = \tilde{N} = \sqrt{\frac{2L}{a} / \ln\left(\frac{\epsilon_0}{T}\sqrt{\frac{a}{L}}\right)}, \tag{7}$$

$$\epsilon = \tilde{\epsilon} = \tilde{N}T. \tag{8}$$

Here

$$\ln\sigma = -2\sqrt{\frac{2L}{a}\ln\left(\frac{\epsilon_0}{T(L/a)^{1/2}}\right)} = -Q_0. \tag{9}$$

For the conductivity mechanism proposed by Pollak and Hauser[2] to be in effect, the surface area S of the sample must be reasonably large, i.e., the following condition must be satisfied: $S > S_0 \exp[\Omega(\tilde{N},\tilde{\epsilon})] = S_c$, where $S_0 \sim (L/\tilde{N})^2$ is the area corresponding to a single chain. The physical meaning of this condition is that there should be at least one optimum chain within the limits of the surface area of the sample.

We show here that if the surface area of the sample is small ($S \ll S_c$), the random oscillations are described by the quantity $\ln\sigma$ as the temperature is varied. If $S \ll S_c$, the flow of current through the sample is determined by several maximally conducting chains of the chains contained in the given sample. In this case, $\ln\sigma$ will fluctuate from one sample to the next by an amount on the order of unity. This situation was analyzed for the first time in Ref. 3, where the transparency of a finite system of independent filaments with impurities was studied. We will calculate a typical value of the conductivity logarithm. We introduce the parameter

$$\nu = \frac{\ln(S/S_0)}{\Omega(\tilde{N}, \tilde{\epsilon})} = \frac{2\ln(S/S_0)}{Q_0}, \tag{10}$$

so that $S \ll S_c$ corresponds to $\nu < 1$. The parameters N and ϵ of the chains which

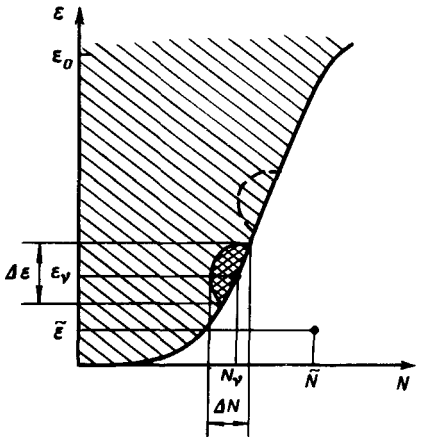

FIG. 1.

determine the flow of current are related by

$$\frac{\nu Q_0}{2} = \Omega(N, \epsilon) = N \ln \frac{\epsilon_0}{\epsilon} . \tag{11}$$

This relation implies that a typical sample with an area S typically has a single chain. Relation (11) corresponds to the boundary of the hatched region in Fig. 1. In a typical sample, there are no chains with parameters N and ϵ that lie outside this region. The parameters N_ν and ϵ_ν of the maximally conducting chains are determined by minimizing $u(N,\epsilon)$ and by imposing additional condition (11)

$$\epsilon_\nu = \tilde{\epsilon}/\nu = T\tilde{N}/\nu , \tag{12}$$

$$N_\nu = \nu \tilde{N} . \tag{13}$$

For the conductivity logarithm we find

$$\ln\sigma = \ln\left[\frac{S_0}{S} e^{-u(\epsilon_\nu, N_\nu)}\right] = -\frac{Q_0}{2}\left(\nu + \frac{1}{\nu}\right). \tag{14}$$

The chains with the parameters N and ϵ, which lie in the small region near the point N_ν, ϵ_ν, actually contribute to the conductivity. (The cross hatching represents this region in Fig. 1.) The scale dimensions of this region are determined from the condition $[U(N,\epsilon) - u(N_\nu, \epsilon_\nu)] \sim 1$, which corresponds to

$$\Delta\epsilon = T\sqrt{\frac{Q_0}{\nu \ln(\epsilon_0/\tilde{\epsilon})}} , \tag{15}$$

$$\Delta N = \sqrt{\frac{Q_0 \nu^3}{\ln^3(\epsilon_0/\tilde{\epsilon})}} . \tag{16}$$

We should point out now that the quantities ϵ_ν and N_ν are temperature dependent. As the temperature is varied, the region of the parameters of the chains that contribute to the conductivity shifts, together with E_ν and N_ν, along the curve. If the temperature is varied by an amount on the order of

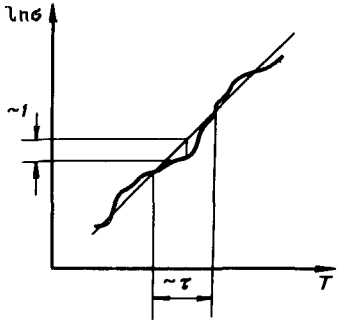

FIG. 2.

$$\tau = T\sqrt{\frac{a}{L}\ln\frac{s}{s_0}} = 2T\sqrt{\frac{\nu}{Q_0}\ln\frac{\epsilon_0}{\tilde{\epsilon}}}\ , \tag{17}$$

the shifted region (the dashed curve in Fig. 1) will no longer overlap the original region, so that there is a complete replacement of chains that contribute to the conductivity, which is equivalent to a change in the impurity configuration in the sample. In other words, it is equivalent to switching to another sample. The temperature dependence of the conductivity logarithm of the sample with a small area is shown schematically in Fig. 2. A typical oscillation period is on the order of $\tau \ll T$ and the oscillation amplitude is on the order of unity (at $\nu \lesssim 1$), in agreement with the characteristic scatter in the values of $\ln\sigma$ in the samples. Varying the temperature by an amount corresponding to τ changes the average value of $\ln\sigma$ by an amount on the order of $[Q_0\nu/\ln^3(\epsilon_0/\tilde{\epsilon})]^{1/2} \gg 1$, so that the oscillations are not "curved." Such a behavior of the conductivity (an effective change in the configurations in a single sample resulting from a change in the external parameter) is analogous to the mesoscopic effects which were studied in Refs. 4–6.

We present some numerical estimates. For amorphous silicon we have[1] $a = 3$ Å and $g = 3\times 10^{19}$ eV^{-1}·cm^{-3}. At $T = 77$ K the maximum thickness L of the film at which the chains still determine the current flow is $L = 700$ Å. In this case we have $\Omega(\tilde{N},\tilde{\epsilon}) = 21$, so that the maximum linear dimension of the sample which allows the oscillations described above to occur is $\sqrt{s_c} \sim 10$ μm. The characteristic oscillation period is $\tau \sim 10$ K.

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