Possible number of different neutrinos

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The number of different types of neutrinos may be related to the dimensionality of our space. This hypothesis is illustrated by an old model (dating from 1955) of a four-dimensional relativistic oscillator in which "spirits" are exorcised by an additional condition analogous to the Virasoro conditions (1970) for a string or, in another version, by an additional term in the equation itself.

We would like to discuss the possibility of a relationship between the number of neutrinos which exist in nature with the dimensionality of the space, and we would like to offer a hypothesis regarding the physical meaning of the quantum number (lepton charge) which distinguishes the different types of neutrinos. Astrophysical data (Ref. 5, for example) have led to the opinion that there can be no more than four or five types of neutrinos in nature. In this letter we illustrate the relationship between this limitation and the dimensionality of the space by means of the model of a relativistic four-dimensional oscillator which is an element of a relativistic string, a generalization of the Dirac equation, which was proposed in Refs. 1 and 2 through the use of internal degrees of freedom. Working on the basis of the oscillator models discussed in those papers, we consider the low-lying excitations which arise in these models to be neutrinos. Following Refs. 1 and 2, we write an equation for the Fourier component of the wave function $\Psi(x^\mu,\xi^\mu)$, where x^μ are the coordinates of the center of mass of the particle in our four-dimensional space-time, and for the bispinor $\Psi(k^\mu\xi^\mu)$ ($\mu = 0,1,2,3$). Here the energy-momentum 4-vector k^μ describes the motion of the center of mass of the structured particle, and the internal coordinate, the 4-vector ξ^μ, describes the "relative" oscillatory motion of the relativistic oscillator (the relativistic string consists of a set of oscillators):

$$\left\{ i\gamma_\mu k^\mu + m_0 - \frac{a}{2}\left[-\frac{\partial^2}{\partial\xi_\mu \partial\xi^\mu} + \xi_\mu \xi^\mu \right] \right\} \Psi = 0, \tag{1}$$

An auxiliary relativistic condition which eliminates oscillatory states along the time axis ("spirits")[1,2] is imposed on Ψ:

$$k^\mu\left(\xi_\mu + \frac{\partial}{\partial\xi^\mu}\right)\Psi = 0. \tag{2}$$

We describe a neutrino as a plane wave [a solution of Eq. (1)] with auxiliary condition (2)] with wave vector k^μ; the mass of a particle is determined in the usual way ($k^\mu k_\mu = m^2$). In 1970, Virasoro[3] also discussed an analog of auxiliary condition (2) for a string; that condition eliminated "spirits," in this case oscillatory states constituting a string of oscillators along the time axis.

For the time-like vector k^μ, solutions of system (1), (2) which were normalized in terms of the internal variables were found in Ref. 1 through a transformation to the center-of-mass frame ($\mathbf{k}=0$), and a growing spectrum was constructed. That spectrum was the mass spectrum of a three-dimensional oscillator: $m = M + \alpha(n_1 + n_2 + n_3)$, $n_i = 0,1,2,\ldots$. It can be shown that there are no solutions of system (1), (2) with space-like and light-like vector k^μ. In the oscillator model (1) with auxiliary condition (2), there can thus be four different neutrinos: one of mass M (the ground state) and three with the identical mass $M+\alpha$. The differences among these three neutrinos are described by the differences in the vibrational states of the oscillator (n_1,n_2,n_3), i.e., (1,0,0), (0,1,0), (0,0,1). The indices—lepton charges—assigned to the different neutrinos thus have the meaning of vibrational states of the structure variable ξ^μ in this approach. According to this model, there is yet another possibility, if the coefficient α in (1) is equal to $-m_0$. In this case, there might be a state with a zero mass for all indices $n_i = 0$, but it would be forbidden by auxiliary condition (2), which eliminates spirits. Consequently, according to Eqs. (1) and (2) with a special coupling of the coefficients m_0 and α, there exist only three neutrinos (with identical mass)—corresponding to the dimensionality of the space. These three neutrinos differ in vibrational states. Transitions among the four neutrinos (in the former case) or among the three (in the latter case) do not occur according to this model.

We would like to discuss yet another model, proposed in Refs. 1 and 2, in which auxiliary condition (2) is replaced by an elimination of the spirits by means of an additional term in the Dirac equation with internal variables. This exorcism of the spirits is analogous in a rather distant way to a string theory proposed by Chang and Monsouri[4] in 1972. Following Refs. 1 and 2, we write an equation for the bispinor $\Psi(k^\mu,\xi^\mu)$ in the form

$$\left\{ i\gamma_\mu k^\mu + m_0 - \frac{a}{2}\left[-\frac{\partial^2}{\partial\xi_\mu\partial\xi^\mu} + \xi^\mu\xi_\mu + \frac{\left(k^\mu \frac{\partial}{\partial\xi^\mu}\right)^2 - (k^\mu\xi_\mu)^2}{k_\mu k^\mu} \right] \right\}\Psi = 0. \quad (3)$$

It can be shown that this equation has no solutions corresponding to a space-like energy-momentum vector k^μ, while a state with a time-like vector k^μ can be found by transforming to the center-of-mass frame ($\mathbf{k}=0$). The mass spectrum is described by the spectrum of a three-dimensional harmonic oscillator, as in the preceding model. In this model, there are four neutrinos, one of mass M and three of identical mass $M+\alpha$. A distinctive feature of this equation is the possibility of a solution with a light-like energy-momentum vector (a zero-mass solution); although this equation has such a solution, its interpretation is not clear at this point. This solution is normalized in terms of one of the internal variables $(\xi^0 - \xi^1)$, and it is independent of the other three. The number of possible neutrinos is therefore again related to the dimensionality of the space and is equal to four. Finally, we consider yet another possibility, based on the model of a relativistic oscillator proposed in Refs. 1 and 2:

$$\left\{ i\gamma_\mu k^\mu + m_0 - \frac{a}{2}\left[-\frac{\partial^2}{\partial\xi_\mu\partial\xi^\mu} + \xi^\mu\xi_\mu + 2\frac{\left(k^\mu \frac{\partial}{\partial\xi^\mu}\right)^2 - (k^\mu\xi_\mu)^2}{k_\mu k^\mu} \right] \right\}\Psi = 0. \quad (4)$$

The additional term with the coefficient of 2 leads to a change in sign in the mass spectrum on the term related to vibrations along the time axis. Equation (4) thus has a growing mass spectrum, which is described by the spectrum of a four-dimensional harmonic oscillator. We thus have five neutrinos, one of mass M and four of mass $M+\alpha$. According to Eqs. (3) and (4), there are no transitions between neutrinos (there are no oscillations). If we take into account the interaction of the relativistic oscillator (the coordinate ξ^{μ}) with a heat bath, we find that there can be transitions from higher-lying levels to lower-lying levels (the neutrinos will have lifetimes). According to the model with the auxiliary condition, there is either one stable neutrino or three stable neutrinos in this case. According to models (3) and (4), there is one stable neutrino and there are three or four unstable neutrinos, respectively.

In summary, the dimensionality of the space imposes a limitation on the possible number of different neutrinos. The total number of different neutrinos may vary from three to five (depending on the model), in agreement with astrophysical data. According to the first two of the models discussed here, the states of different neutrinos transform under irreducible representations of the SU_3 group, while according to model (4), they transform under representations of the SU_4 unitary group. A more-detailed paper on the subject will be published separately.

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