Lifetime of macroscopic current states

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The determination of the rate Γ at which the current states decay in a Josephson junction should take into account the effect of quantum fluctuations on the "current-phase" dependence of such a junction. Analysis of the temperature dependence of Γ shows that Γ does not depend on the capacitance of the junction in the strong-dissipation limit if the quantum correction to the current is taken into account. The results obtained in this study are in quantitative agreement with the results obtained experimentally by Schwartz *et al.* [Phys. Rev. Lett. **55**, 1547 (1985)].

Several interesting phenomena which are currently being studied extensively both theoretically and experimentally are linked with the quantum fluctuations in the macroscopic systems. One such phenomenon involves the damping of the current states of the superconducting contacts and SQUID's. The rate at which the quantum decay of the states of this sort occurs is exponentially small, $\Gamma = B_0 \exp(-A_0)$, where $A_0 \gg 1$. In several experiments a quantitative agreement (within quasiclassical accuracy) with the theoretical expressions for the quantity $\ln\Gamma$ has now been reached in various limiting cases.

In recent experiments[1] an attempt has been made for the first time to determine the coefficient of the exponential function B_0. The theoretical value of B_0 (for the parameters used in Ref. 1) is five orders of magnitude higher than the experimental value of the coefficient of the exponential function B_0. The theoretical value of B_0 (for the parameters used in Ref. 1) is five orders of magnitude higher than the experimental value of the coefficient of the exponential function obtained experimentally in Ref. 1, whereas the results of the measurements of Ref. 1 for the temperature dependence of the argument of the exponential function $\ln\Gamma$ are in excellent agreement with theory.

In the present letter we show that this discrepancy is directly connected with the quantum renormalization of the critical current of the superconducting contacts which was predicted earlier by Zaikin and Panyukov.[2] If this discrepancy is taken into account, the rate of the decay Γ will be in complete agreement with the results of Ref. 1. We will analyze the temperature dependence of the coefficient of the exponential function B.

The self-energy of the weak superconducting couplings can be determined by using the functional integral

$$F = -T\ln\int D\varphi \exp\{-S[\varphi]\}, \tag{1}$$

where T is the temperature, and 2φ is the phase difference at the contact. The microscopic calculation of the action S for various types of superconducting contacts was carried out in Refs. 3–5. The expression for the free energy in the classical theory immediately follows from (1) if the fluctuation of the phase φ is ignored. In the adiabatic approximation we obtain the following expression by the method of steepest descent:

$$F_0 = V(\varphi_0) - I\varphi_0/e, \qquad I/e = \partial V(\varphi_0)/\partial\varphi_0, \tag{2}$$

where $V(\varphi)$ is the potential, and I is the external current flowing through the contact. The classical critical current $I = I_{c0}$ is found from the condition for the loss of the continuous solution of the second equation in (2), $V''(\varphi_{c0}) = 0$.

We assume that $I_{c0} - I \ll I_{c0}$ and that φ_{c0} is not too close to $\pi/2$. Under these conditions we can make use of the condition $\varphi_1 \equiv \varphi - \varphi_0 \ll 1$ and retain only the dominant terms of the expansion of S in powers of φ_1. In the single-loop approximation a calculation of the functional integral (1) will then yield (see Ref. 2)

$$F = F_0 + \frac{\kappa}{2}\varphi_1^2 - \frac{\lambda}{6}\varphi_1^3 + T\ln\left[\frac{\Gamma(a_+(\kappa))\Gamma(a_-(\kappa))}{\Gamma(a_+(\kappa_1))\Gamma(a_-(\kappa_1))}\right], \tag{3}$$

where $\kappa = [(2\lambda/e)(I_{c0} - I)]^{1/2}$, $\lambda = -\partial^3 V(\varphi_0)/\partial\varphi_0^3$, $a_\pm(\kappa) = \frac{1}{2} + (\eta \pm \sqrt{\eta^2 - 4m\kappa})/4\pi m T$, $\kappa_1 = \kappa - \lambda\varphi_1$, $\Gamma(x)$ is the Euler gamma function, $e^2 m = C^*$ is the renormalized capacitance of the contact,[4,5] and $(e^2\eta)^{-1} = R_{\rm eff}$ is the shunt resistance[4] or the effective short-circuit resistance.[5] The equilibrium value $\varphi_1 = \langle\varphi_1\rangle$ can be found by minimizing the functional (3) (see also Ref. 5). The equation which determines the "current-phase" dependence for the superconducting contact differs in this case from the second equation in (2). In the case of the tunnel contact ($\partial V/\partial\varphi = e^{-1}I_{c0}\sin 2\varphi$) we find the following expression with the help of (3) ($\langle\varphi\rangle = \varphi_0 + \langle\varphi_1\rangle$):

$$I = I_{c0}\sin 2\langle\varphi\rangle - \frac{e\lambda}{2}\beta(\kappa_1(I)), \qquad \beta(\kappa) = \frac{\psi(a_+(\kappa)) - \psi(a_-(\kappa))}{\pi\sqrt{\eta^2 - 4m\kappa}}, \tag{4}$$

where $\psi(x) = \Gamma(x)/\Gamma(x)$. The expression for the quantum correction to the current [expression (4)] is valid if $I_{c0} - I \ll I_{c0}$. A break in the continuous solution of (4) at the point[2] $I = I_Q \equiv I_{c0} - e[\lambda\beta(\kappa_c) + \kappa_c^2/\lambda]/2$, $\kappa_c = -\lambda^2\beta(\kappa_c)/2$ means that at

$I<I_Q$ there is no time-independent average value (with allowance for the quantum flutuations) of the field φ (or φ_1) and that at $I<I_Q$ the probability for the existence of an average field is nonzero.

We consider here the most interesting limit of strong dissipation $\eta^2 \gg m\kappa$. We see from (4) that the dependence of the magnetic flux ϕ in a SQUID on the external flux ϕ_x differs from the well-known formula for $\phi(\phi_x)$ for a sinusoidal current-phase dependence. At $T=0$, incorporating (4), we find

$$I_c \equiv (\phi_x - \phi)\left(L \sin\frac{2\pi\phi}{\phi_0}\right)^{-1} = I_{c0} - \frac{e\lambda}{2\pi\eta} \ln\frac{\eta^2}{m\kappa_1} , \tag{5}$$

where L is the inductance of the SQUID, and ϕ_0 is a fluxoid. The current I_c ($<I_{c0}$) introduced into (5) is a directly measurable quantity. The current I_c should not be given the same meaning as the current I_{c0} in the classical theory, since $\kappa_1 = \kappa_1(I)$ and hence $I_c = I_c(\phi_x)$. Despite the fact that the quantum correction is small, $\delta\phi \equiv L(I_{c0} - I_c)$, it was in fact this correction that was measured in the precision experiments in Ref. 1. For the values used in Ref. 1, $LI_c \simeq 4\phi_0$, $R_{\rm eff} \simeq 9\Omega$, and $\eta^2/m\kappa_1 \simeq 10^2$, we find, with use of (5), $\delta\phi \simeq 0.025\phi_0$. This result is in excellent agreement with the value measured in Ref. 1, $\delta\phi = (0.027 \pm 0.010)\phi_0$.

The difference between I_c and I_{c0} should be taken into account in determining the decay rate of the metastable current states. At $\eta^2 \gg m\kappa$ the expression for the argument of the exponential function A_0 was obtained by Caldeira and Leggett[6] (at $T=0$) and by Larkin and Ovchinnikov[4,7] (at a nonzero temperature). The expression for B_0 was obtained in Ref. 7 . In each of the above cases the equation $\Gamma = 2{\rm IM}F$ was used to calculate Γ. At $T \ll T_{c0} = \kappa/2\pi\eta$ we find

$$A_0 = \frac{4\pi\eta}{e\lambda}(I_{c0} - I) - \frac{(2\pi\eta)^3}{3\lambda^2} T^2 , \qquad B_0 = \frac{\sqrt{2}\eta^{7/2}}{\lambda m^2} . \tag{6}$$

The expression for B_0 diverges as $m \to 0$. A divergence of this sort can be eliminated by expressing I_{c0} in (6) in terms of I_c measured experimentally. We should bear in mind that I_c depends on the current I^* (or the flux $\phi_x{}^*$) and on the temperature T^* at which this current (I_c) is measured [see Eq. (4)]. We finally find ($T^* \ll \eta/m$)

$$\Gamma = B\exp(-A), \quad B = \frac{2^{-5/2}(\pi T^*)^2 \eta^{3/2}}{\lambda} \exp\left[2\psi\left(\frac{1}{2} + \frac{T_c}{T^*}\right)\right], \quad T \ll T_c , \tag{7}$$

where $\kappa_1{}^* = \kappa_1(I^*)$, $T_c = \kappa_1{}^*/2\pi\eta < T_{c0}$, and A is determined by the equation for A_0 in (6) in which I_{c0} should be replaced by I_c. At $T^* \ll T_c$ we have

$$B = \sqrt{2}\eta^{3/2}\kappa_1^{*2}\lambda^{-1} . \tag{8}$$

For the parameters used in the experiments in Ref. 1, the quantity B in (8) is 4–5 orders of magnitude lower than B_0 in (6), also in good agreement with the results of Ref. 1. In the region $T_c < T \ll \eta/m$, at $T^* \ll \eta/m$, we have

$$\Gamma = B\exp(-V_c/T), \qquad V_c = \frac{4}{3}\sqrt{\frac{2}{e^3\lambda}}(I_c - I)^{3/2},$$
$$B = B_0\left(\frac{2\pi T^* m}{\eta}\right)^{2T_c/T} \exp\left[\frac{2T_c}{T}\psi\left(\frac{1}{2} + \frac{T_c}{T^*}\right)\right] < B_0. \tag{9}$$

The value of B_0 for the activation decay (9) has been calculated in several studies (see, e.g., Refs. 7 and 8). A decrease in B in comparison with B_0 has also been observed experimentally at $T > T_c$ in Ref. 1. Accordingly, the discrepancy between theory and experiment, which has been pointed out above, will be absent completely if the difference between the measured value of the current I_c and the seed value of the current I_{c0}, which stems from the quantum fluctuations, is taken into account. The conditions under which I_c is measured are important. If, for example, independent measurements of I_c are carried out at temperatures at which Γ is measured, we can assume that $T^* = T$ in Eqs. (7) and (9).

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