Quantum spin glasses in the Ising model with a transverse field

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The paramagnet–spin-glass phase transition in the Ising model with a transverse field is considered. The transverse field can stabilize the paramagnetic state, in spite of the strong quantum fluctuations in the system.

The quantum fluctuations of the order parameter are usually unimportant in the case of second-order phase transitions in crystals. The quantum corrections to all thermodynamics quantities, which are proportional to r_0^{-3} (r_0 is the interaction radius), vanish in the limit $r_0 \to \infty$, just as the classical correlation effects. If r_0 is on the order of the lattice constant, the quantum corrections will have numerically small values.

The situation is completely different in the case of spin glasses, in which neither the classical nor quantum correlation effects vanish even in the infinite-interaction-radius model (the Sherington-Kirkpatrick model). The classical correlation effects, which are generally of crucial importance, are responsible for the instability of the ergodic phase and for the transition to spin glass. As the study of the Heisenberg quantum model[1] has shown, the quantum fluctuations suppress the spin-glass state rather strongly. In particular, they lower the transition temperature T_f by a factor of approximately two.

Experiments on the so-called proton glasses,[2,3] which are a mixture of ferroelectrics and antiferroelectrics, have recently generated several studies[4,5] on spin glasses in the Ising model with a transverse field, whose Hamiltonian is

$$\mathcal{H} = -\Delta \sum_i S_{iz} - \sum_{(ij)} J_{ij} S_{ix} S_{jx} . \qquad (1)$$

Pirc *et al.*[4] assumed the spin to be a classical spin, and Ishii and Yamamoto[5] studied the quantum model with $S = 1/2$. This particular model describes the proton glasses. We know that in an ordered magnetic material with Hamiltonian (1) a sufficiently strong transverse field, $\Delta > \Delta_c$, destroys the order even at $T = 0$. The situation is similar in the classical model with a random (in terms of the sign) interaction J_{ij}: A strong field breaks off the transition[4] to spin glass.[1)] Using the Thouless-Anderson-Palmer method,[6] Ishii and Yamamoto[5] developed a theory of perturbation in Δ/T for the quantum model. The corrections to the free energy in the paramagnetic phase and the corrections to T_f have been calculated. Having analyzed the structure of several perturbation theories, Ishii and Yamamoto[5] concluded that the properties of the quantum model are fundamentally different from those of the classical model: The quantum model always gives rise to a spin glass at low temperatures, regardless of the strength of the field.

In the present study we derive an expression for the free energy and for T_f in a field of arbitrary strength. We will show that the assertion made by Ishii and Yamamoto[5] is incorrect; i.e., the quantum fluctuations do not stabilize the spin-glass state in the model under consideration.

In the method of replicas, the free energy can be found by averaging the expression

$$\beta F = -\lim_{n \to 0} \frac{1}{n} [\mathrm{Tr}\{ e^{-\beta \mathcal{H}_0} T \exp (\beta \int_0^1 d\tau \sum_{(i,j)} J_{ij} \sum_{\alpha=1}^{n} S^{\alpha}_{ix}(\tau) S^{\alpha}_{jx}(\tau)) \} - 1] \tag{2}$$

over the distribution J_{ij} which is assumed to be a normal distribution with a zeroth mean value and a dispersion $J/\sqrt{N}$, where N is the total number of spins. Here $\beta = T^{-1}$, τ is the apparent time, $\mathcal{H}_0 = -\Delta \Sigma_{\alpha,i} S^{\alpha}_{iz}$, and $S(\tau)$ are the operators in the interaction representation. An average over J_{ij} can be taken in the usual way, since the operators can be permuted in the T-product. Following Ref. 1, we can then transform the expression, found after taking an average, into a functional integral in the fields $y^{\alpha\beta}(\tau,\tau')$:

$$\beta F = -\lim_{n \to 0} \frac{1}{n} [S \prod_{(\alpha,\beta)} Dy^{\alpha\beta}(\tau,\tau') \prod_{\alpha} Dy^{\alpha\alpha}(\tau,\tau') e^{-N\Phi} - 1],$$

$$\Phi = \int_0^1 \int_0^1 d\tau d\tau_1 \left[\frac{1}{2} \sum_{(\alpha,\beta)} (y^{\alpha\beta}(\tau,\tau'))^2 + \sum_{\alpha} (y^{\alpha\alpha}(\tau,\tau'))^2 \right]$$

$$- \ln \mathrm{Tr}\{ e^{-\beta J/0} T \exp[\beta J \int_0^1 \int_0^1 d\tau d\tau' (\sum_{(\alpha,\beta)} y^{\alpha\beta}(\tau,\tau') S^{\alpha}_x(\tau) S^{\beta}_x(\tau')$$

$$+ \sum_{\alpha} y^{\alpha\alpha}(\tau,\tau') S^{\alpha}_x(\tau) S^{\alpha}_x(\tau'))] \}. \tag{3}$$

We calculate integral (3) according to the method of steepest descent. In the high-temperature phase, all the functions $y^{\alpha\beta}(\tau,\tau')$ which satisfy the equations for the steepest descent vanish and for the functions $y^{\alpha\alpha}(\tau,\tau')$, which do not depend on the replica index at any temperature, we find

$$R(\tau,\tau') = \frac{2}{\beta J} y^{\alpha\alpha}(\tau,\tau') = \langle T S_x(\tau) S_x(\tau') \rangle . \tag{4}$$

An average in (4) is taken with the Hamiltonian $H_0 + H_{\mathrm{eff}}$, where

$$\mathscr{H}_{\text{eff}} = -\frac{1}{2}\beta J^2 \int_0^1\int_0^1 d\tau\, d\tau' R(\tau,\tau') S_x(\tau) S_x(\tau') . \tag{5}$$

To determine the transition point, we expand F in $y^{\alpha\beta}(\tau,\tau')$

$$\frac{\beta F}{N} = \frac{\beta^2 J^2}{4} \sum_{\omega_n} R^2(\omega_n) - \ln \operatorname{Sp} e^{-\beta(\mathscr{H}_0 + \mathscr{H}_{\text{eff}})}$$

$$+ \lim_{n\to 0} \frac{1}{4n} \sum_{\omega_n} \sum_{(\alpha\beta)} (y^{\alpha\beta}(\omega_n))^2 [1 - (\beta J)^2 R^2(\omega_n)] . \tag{6}$$

In (6) the summation is over the discrete frequencies $\omega_n = 2\pi n$. We easily see that $R(\omega_n)$ decreases with increasing n. The equation for T_f can therefore be written as

$$T_f = JR(\omega_n = 0). \tag{7}$$

If the field Δ is small, $\Delta/J \ll 1$, then by expanding in Δ/T in (4) and (5) and substituting the result of the expansion in (7), we find

$$T_f = \frac{J}{4}\left(1 - 32\frac{\Delta^2}{J^2}\int_0^1 d\tau \int_0^1 d\tau' \int_0^1 d\tau_1 \int_0^1 d\tau_2 \left[< TS_z(\tau) S_z(\tau') > \frac{1}{4} - < TS_z(\tau) S_z(\tau') S_z(\tau_1) S_z(\tau_2) > \right]\right). \tag{8}$$

An average in (7) is taken with the Hamiltonian H_{eff}, in which we can set $R(\tau,\tau') = 1/4$. Transforming the expectation values in (8) with the help of the Hubbard-Stratonovich identity, we then finally find

$$T_f = \frac{J}{4}[1 - 16(\Delta/J)^2 \int_0^1 x^2(1 - x) \exp\{-2x(1-x)\}\, dx], \tag{9}$$

consistent in terms of our notation with the result of Ref. 5.

Let us now consider the strong fields and low temperatures $T \to 0$. Expanding $R(\tau,\tau')$ in (4) in a series in H_{eff}, we easily see that in the limit $T \to 0$, this expansion is in the parameter J/Δ. This means that at $\Delta/J \gg 1$ we can confine the analysis to the first term of this expansion and

$$R(\omega_n) = \frac{1}{2}\frac{\Delta T}{\omega_n^2 + \Delta^2} .$$

At $\Delta/J \gg 1$ the right side of Eq. (7) is therefore always smaller than T, indicating that the paramagnetic solution is stable to within $T = 0$.

Ishii and Yamamoto[5] have erroneously concluded that a phase transition to a spin glass can occur at large values of Δ/J because of an unjustifiable extrapolation of an equation of the type in Ref. 8, which is valid only in perturbation theories under the condition $\Delta \ll J \sim T_f$, to the limit $T \to 0$.

[1)]The result and the qualitative phase diagram obtained by Pirc *et al.*[4] are correct, but the equations for the symmetric solution derived by them are incorrect.

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