Harmonic analysis on a Lorentz group and the proton form factor

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(Submitted February 17, 1977)
Pis'ma Zh. Eksp. Teor. Fiz. **25**, No. 7, 334–337 (5 April 1977)

It is shown that in terms of the relativistic relative coordinate, the vector mesons contribute to the spatial structure of the proton only at the distances larger than its constant wavelengths. Allowance for the contribution of the central region of the proton leads to an expression that describes adequately the experimental data.

PACS numbers: 13.40.Fn, 12.40.Vr, 14.20.Ei, 11.30.Cp

It was proposed in[1] to describe the spatial structure of particles in terms of an invariant distribution $F(r)$, the role of the Fourier transformation in the transition to which is played by expansion of the form factor $F(t)$ in unitary irreducible representations of the Lorentz group[2,3]

$$F(t) = 4\pi \int_0^\infty \frac{\sin rMy}{rM\,\mathrm{sh}y} F(r) r^2 dr \,. \tag{1}$$

Here M is the mass of the proton, and the hyperbolic angle y is the rapidity corresponding to the momentum transfer $t=(p-k)^2$, namely $y=\cosh^{-1}(2M_t^2/2M^2)$.

The modulus of the relativistic relative coordinate r introduced with the aid of (1) is a relativistic invariant. The expression for the invariant mean squared radius (MSR) of the particle is expressed in terms of $F(r)$ as follows[1]:

$$< r_0^2 > \equiv 6 \frac{\partial F(t)}{\partial t}\Big|_{t=0} \Big| F(0) = \frac{1}{M^2} + \frac{\int r^2 F(r) dr}{\int F(r) dr} = \frac{1}{M^2} + < r^2 >. \tag{2}$$

With the aid of (2) it is easy to verify that a central sphere with $\langle r_0^2\rangle = 1/M^2$ corresponds to the function $F(r)=\delta(r)/4\pi r^2$, which leads in accordance with (1) to the following value of the contribution from this sphere to the particle form factor

$$F(t)\Big|_{r_0=1/M} = \frac{\sin rMy}{rM\,\mathrm{sh}y}\Big|_{r=0} = \frac{y}{\mathrm{sh}y} = \frac{\mathrm{Arch}\dfrac{2M^2-t}{2M^2}}{\sqrt{t(t-4M^2)}} \,. \tag{3}$$

The factor $y/\sinh y$, together with the corresponding central region with $\langle r_0^2\rangle = 1/M^2$, has no nonrelativistic analogs, inasmuch as $\hbar/MC \to 0$ and $y/\sinh y \to 1$.

The proton region described by the contribution of the vector mesons was analyzed in[1] from the point of view of the new coordinate space. The transform $F(r)$ of the meson propagator $1/(\mu^2 - t)$ is a constant sign function at $\mu^2 < 4M^2$ and an oscillating function at $\mu^2 > 4M^2$.[1,2] Inasmuch as the masses of the hitherto discovered vector mesons ρ, ω, ϕ, and ρ'' (1550) satisfy the inequality

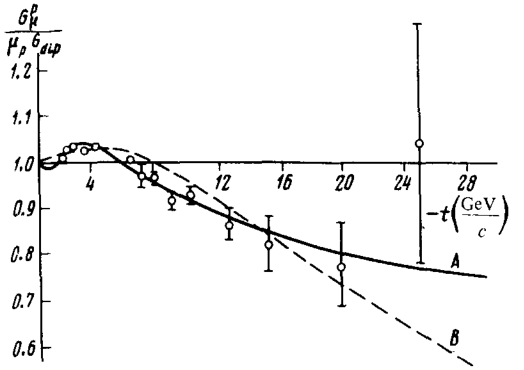

FIG. 1. Curve A corresponds to the form-factor parametrization given by the VDM modified at small distances; curve B corresponds to a fit to the formulas of the ordinary VDM (the data were taken from[5]).

$\mu_V^2 < 4M^2$, it follows that the function $F(r)$, which describes the contribution of the mesons to the proton structure, is of constant sign, and the corresponding quantity $\langle r^2 \rangle_p$ is positive. Consequently, these vector mesons produce the proton structure at distances larger than its Compton wavelength.[1)]

To map the entire proton structure in momentum space, with allowance for the contributions from the ρ, ω, ϕ, and ρ'' mesons in accordance with the VDM, it is necessary to add the contribution of its central part. As a result, the formula for the electromagnetic form factor of the proton becomes

$$F_p(t) = \frac{y}{\operatorname{sh} y} \sum_V \frac{a_V}{\mu_V^2 - t} . \tag{4}$$

With the aid of (3) it is easily seen that formula (4) has a correct "almost-dipole" asymptotic behavior

$$F_p(t) \underset{|t| >> M^2}{\rightarrow} \frac{\ln |t|/M^2}{|t|^2} . \tag{5}$$

In the region of small momentum transfers $|t| < 1$ (GeV/c)^2, the factor $y/\sinh y \approx 1$, i.e., the use of the pure VDM is valid.

According to the quark-model estimates,[4] the relative motion of the e-

TABLE I.

№	Model	χ_F^2 with contribution of the central part		χ_F^2 without contribution of the central part		Number of free parameters
		Data 5, 14 points	Data 6, 84 points	Data 5, 14 points	Data 6, 84 points	
1	4 poles $(\rho, \omega, \phi, \rho'')$	0.67	1.04	1.04	1.78	3
2	VDM$(\rho, \omega, \phi, \rho', \rho'')$	0.81	1.90	2.05	3.61	3
3	VDM with "core" $(\rho, \omega, \phi, \rho')$	0.76	0.88	1.90	1.12	4
4	VDM with "core" $(\rho, \omega, \phi, \rho'')$	0.69	0.92	0.89	1.31	4

quarks making up the proton is confined to a region with dimension equal to the Compton wavelength. It can consequently be assumed that it is they which govern the contribution of the central part of the proton (3). The quark-antiquark pairs form in this case vector mesons, and it is these which produce, at $\mu_V^2 < 4M^2$, the structure at distances larger than the Compton wavelength of the proton.

We have compared the experimental data on the proton magnetic form factor[5,6] with predictions based on formula (4). We use also a different frequently employed parametrization, corresponding to the possible presence of a contribution from the "core." (In our approach, the role of the "core" is played by the central region with $\langle r_0^2 \rangle = 1/M^2$ and $F(t) = y/\sinh y$)

$$F(t) = \frac{y}{\operatorname{ch} y}\left[\left(1 - \sum_V \frac{a_V}{\mu_V^2}\right) + \sum_V \frac{a_V}{\mu_V^2 - t}\right]. \tag{6}$$

The result of the data reduction are summarized in Table I, which shows for comparison the values of χ^2 per degree of freedom, χ_F^2, both in accordance with formulas (4) and (6) and in accordance with the formulas of the ordinary VDM. The figure shows the behavior of the proton magnetic form factor for model 1 of Table I.

We see thus that allowance for the contribution from the central region with $\langle r_0^2 \rangle = 1/M^2$ makes it possible, in contrast to[7], to describe quite well the experimental data with allowance for the only discovered ρ, ω, ϕ and ρ'' (1550) mesons. In addition, our approach predicts (see (4)) that the presently observed faster than "dipole" $1/t^2$ decrease of the form factor should give way at asymptotic $|t| \gg M^2$ to a slower decrease, of the type (5). As a result, the curve determined by our model at large $|t|$ should cross the straight line $G_M^P/\mu G_D = 1$.

The authors thank V. G. Kadyshevskiĭ, V. A. Matveev, and V. A. Meshcheryakov for useful discussions and S. M. Eliseev for valuable consultations when working with the computer.

1)In the case of a pion, inasmuch as $\mu_V^2 = \rho, \omega, \phi, \rho' > 4M_\pi^2$, the same vector mesons correspond to oscillating functions $F(r)$ and to negative values of $\langle r^2 \rangle_\pi$, and this leads to the value $\langle r_0^2 \rangle_\pi < 1/M_\pi^2$, in agreement with experiment.

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