Resonant absorption of electromagnetic waves in an inhomogeneous plasma

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(Submitted February 27, 1977)
Pis'ma Zh. Eksp. Teor. Fiz. **25**, No. 8, 355–357 (20 April 1977)

PACS numbers: 52.25.Ps

It is shown that a smooth plasma layer $\epsilon(z)$, in which $[d\epsilon/dz]_{\epsilon=0} \equiv \epsilon_0' < 0$, $[d^2\epsilon/dz^2]_{\epsilon=0} \equiv \epsilon_0'' < 0$ and $|\epsilon_0''| > \epsilon_0'^2$, can absorb an electromagnetic wave of the TM type ($E_z \neq 0$) on account of energy dissipation in the region of the plasma resonance. This effect is due to synchronized excitation of a mode that is quasilocalized in the resonance region.

1. When an inhomogeneous isotropic plasma $\epsilon(z)$ is exposed to a plane wave of the type Tm (E_x, E_z, H_y), energy is absorbed in the region $\epsilon(z_0) = 0$; the character of the absorption is almost independent of the actual loss mechanism.[1,2] The use of this effect for plasma heating is determined by the possibilities of matching the incident field to the plasma. At linear and smooth quasilinear distributions of $\epsilon(z)$, the attainable power absorption coefficients are $Q \lesssim 0.5$. The idea has even been advanced that the value $Q = 0.5$ is the limit. Recent studies[3,4] have established, however, that the presence in the $\epsilon(z)$ distribution of sections with abrupt changes of the permittivity, which admit of the existence of quasilocalized modes, increases Q to unity.

Our present study, stimulated by the experiments of[5], is aimed at the showing that under certain conditions a nonreflecting regime can be obtained also for relatively smooth $\epsilon(z)$ distributions.

2. We use the procedure and terminology employed in the theory of attenuators operating beyond cutoff, since the problem of matching coincides in this case from the mathematical point of view with the corresponding waveguide problem, and the latter has been investigated in sufficient detail. The transverse impedance $Z_\perp$ of the field in the section $z = \text{const}$ is determined from the formula $\mathbf{E}_\perp = Z_\perp [z^0 \times \mathbf{H}_\perp]$ (z^0 is a unit vector along z). The characteristic impedance $\pm Z_0$ is equal to the transverse impedance for a single mode propagating or falling off in the $\pm z$ direction; in the case of inhomogeneous media, the mode is taken to be that for a homogeneous medium with parameters of the given point. The impedance z_0 separates an natural fashion the following regions of an inhomogeneous plasma: (I) the propagation region ($\text{Im} Z_0 = 0, 1 > \epsilon > \gamma^2$; $Z_0 = \sqrt{\epsilon - \gamma^2}/\epsilon$, $\gamma^2 \equiv \sin^2\theta$, and θ is the angle of incidence of the wave from the vacuum), (II) the region of capacitive attenuation ($\text{Im} Z_0 < 0$, $\gamma^2 > \epsilon > 0$), (III) the resonance region ($\text{Im} Z_0 \to \infty$, $\epsilon \sim 0$), and (IV) the region of inductive attenuation ($\text{Im} Z_0 > 0$, $\epsilon < 0$).

It is clear that a nonreflection regime is possible, if at all, only under the condition that the reactive impedances of regions (II) and (IV) cancel each other, i.e., at a unique resonance which, of course, differs from the plasma resonance that leads only to the "swelling" of the field in the vicinity of $\epsilon = 0$. The total-matching effect can therefore be called double resonant absorption; this is in fact a single-model variant of the high-frequency resonant probe.[6]

3. It is easy to obtain the boundary condition for the impedance Z on going through the plane $z=z_0$:

$$Z_\perp(z_0-\Delta z)-Z_\perp(z_0+\Delta z)-R_{\|}(z_0)=0, \tag{1}$$

where $R_{\|}=-\gamma^2(\pi k/\epsilon_0')$ (k is the wave number in vacuum). The resonant-matching regime is characterized by equality of the imaginary parts of $Z_\perp^{(\pm)} \equiv Z_\perp(z_0\pm\Delta z)$, which takes place, in particular, for modes that are quasi-localized near $z=z_0$ with impedances $Z^{(\pm)}$ approximately equal to the corresponding characteristic impedances. Substitution of the latter in (1) leads to the equation

$$\frac{\sqrt{\gamma^2-\epsilon^{(-)}}}{\epsilon^{(-)}}-\frac{\sqrt{\gamma^2+|\epsilon^{(+)}|}}{|\epsilon^{(+)}|}+\frac{i\gamma^2\pi k}{\epsilon_0'}=0, \tag{2}$$

which coincides with the dispersion equation for the surface waves guided by the narrow transition layer.[7] The presence of such transitions ensures in fact the resonant matching,[3,4] but matching can be attained also for smooth distributions. The corresponding equation is obtained from (2) by making the substitution $\epsilon^{(\pm)}=\pm\epsilon_0'\Delta z+\frac{1}{2}\epsilon_0''(\Delta z)^2$, followed by the series expansion

$$\gamma^2+\epsilon_0'^2/\epsilon_0''+i\gamma^3\pi k\epsilon_0'/\epsilon_0''=0. \tag{3}$$

At $k\epsilon_0'/\epsilon_0''\ll 1$ we have for $\gamma_r(\gamma=\gamma_r-i\gamma_{\rm im})$ the following: $\gamma_r^2=-\epsilon_0'^2/\epsilon_0''$.[1] The mode is strictly localized if $\gamma_r^2>1$, i.e., for the layer with $\epsilon_0'<0$, $\epsilon_0''<0$, and $|\epsilon_0''|<\epsilon_0'^2$. For the reverse inequality $|\epsilon_0''|>\epsilon_0'^2$, the mode becomes quasilocalized and can be resonantly excited from the outside by an incident (at an angle $\theta=\theta^*$) plane wave:

$$\sin^2\theta^*=-\epsilon_0'^2/\epsilon_0'' \tag{4}$$

Of course, exact matching depends on a profile on the whole and calls for precise numerical calculations, in which the condition (4)—the necessary matching condition—must serve as the starting point.

4. Nonreflecting layers are best investigated by resorting to the equation for the transverse impedance of a single-mode transmission line with arbitrary characteristic impedance

$$dZ_\perp/dz+ik\epsilon[Z_\perp^2-Z_0^2(z)]=0. \tag{5}$$

Mathematically, the problem consists of selecting a profile $\epsilon(z)$ corresponding to a value of $Z_\perp(z)$ that ensures at the output ($z=0$) the condition of ideal matching: $Z_\perp(0)=Z_0^{(0)}\equiv\cos\theta$. It seems that the simplest way is to make this choice in succession: start with a piecewise-constant approximation, go over to the piecewise-linear approximation, and finally to the approximation that is everywhere smooth; each of these stages constitutes by itself a realizable model. Leaving out the details, we present two examples of smooth nonreflecting profiles obtained numerically with the NAIRI computer assuming constant (independent of z) collisions $\nu_{eff}/\omega = 10^{-4}$ (the latter produces practically no distributed contributions to the matching, but makes possible direct calculation of the field at the point $\epsilon = 0$).

Hyperbolic profile: $\epsilon(z) = \alpha + b(\xi - kz) - \{b^2(\xi - kz)^2 - \frac{1}{2}[(\xi - kz)^2 - \zeta]\}^{1/2}$. Ideal matching (within 1%) is reached at parameter values $\alpha = 0.184$; $b = 5.025$; $\xi = 16.18$; and $\zeta = 0.5$ for an incidence angle $\sin^2\theta^* = \frac{1}{4}$.

Logarithmic profile: $\epsilon(z) = \sin^2\theta \ln\{\alpha[(1/\alpha)\exp(\sin^{-2}\theta) - kz]\}$, in which the condition (4) is satisfied in all the cross sections $-\epsilon'^2/-\epsilon'' = \text{const} = \sin^2\theta$. A nonreflecting regime is realized at $\alpha = 4.2$ for U.

5. Thus, if condition (4) is satisfied, it is possible to construct plasma layers that are ideally matched to incident plane waves of the TM type. In the case of a broad wave beam, the matching is reached only for selected directions, but in the nearer zone of the radiator, this may turn out to be sufficient for the absorption of the entire incident field, as was indeed observed in experiments.[5]

In plasma formations with curved $\epsilon = 0$ surfaces, the condition (4) can be partially satisfied on account of the inhomogeneous metrics, so that matching is possible in principle even for quasilinear profiles $|\epsilon_0''| < \epsilon_0'^2$. It is obvious that the considered effect is reciprocal, i.e., the conditions of total absorption correspond to conditions of optimal emission of sources placed at the section $\epsilon = 0$, including sources of thermal radiation. Finally, although this effect is linear (in the field), it should play an important role also in nonlinear interaction of waves with a plasma, when sections with abrupt changes of ϵ or transitions satisfying the condition (4) can arrive in the self-consistent distribution; the anomalous absorption can take place both at the fundamental frequency and at the probing or modulating neighboring frequencies.

The authors are grateful to Yu. Ya. Brodskiĭ for taking part in the discussions.

1)A transition to a linear layer ($\epsilon'' \to 0$) yields within the framework of (3) the strongly damped solution $\gamma^3 \approx i\epsilon_0'/\pi k$.

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