# Remark on the Fröhlich conductivity in one-dimensional systems

L. P. Gor'kov

*L. D. Landau Institute of Theoretical Physics, USSR Academy of Sciences*
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It is shown, in the limit of arbitrarily small impurity concentrations, that complete pinning of the Fröhlich superstructure takes place in a one-dimensional system, and correspondingly the static conductivity vanishes at $T = 0$.

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In a one-dimensional conducting system, the electron-phonon interaction leads to a structural instability, first noted independently by Peierls and Fröhlich.[1] The instability consists in the development of a lattice deformation of the type $u_0 \cos(2k_F x + \phi)$ because the contribution of the conduction electrons to the elastic energy cancels its lattice component. Accordingly, the total elastic-energy density at zero temperature would be of the form

$$F = \rho \frac{\omega_0^2 u^2(x)}{2} + \frac{2}{\pi v}\left\{ -\frac{\Delta^2}{2} \ln \frac{2E_F}{\Delta} - \frac{\Delta^2}{4} + \frac{v^2}{2}\left(\frac{\nabla\phi}{2}\right)^2 \right\}, \tag{1}$$

where the second term describes the contribution of electrons with a spectrum

$$\epsilon(k) = E_F + vq \pm \sqrt{v^2(k-q)^2 + \Delta^2} \tag{2}$$

and the "gap" $\Delta = |d_1 u_{\pm(2k_F+q)}|$ is due to the lattice deformation:

$$u_{\pm(2k_F+q)} \equiv u^{\pm} = |u^{\pm}| \exp(\pm i(2k_F + q)x \pm i\phi) \tag{3}$$

($d_1$ is the deformation potential of the electrons).

Minimization of (1) with respect to $u^{\pm}$ at $T = 0$, with the definition $\nabla\phi = 0$ [$q = \nabla\phi/2$ in (2)] and with the definition of the deformation

$$\rho\omega_0^2 \Delta - \frac{d_1^2 \Delta}{\pi v} \ln \frac{2E_F}{\Delta} = 0 \tag{4}$$

leaves the choice of the phase free.

Fröhlich[1] advanced the idea that, neglecting defects and umklapp processes, the structure (3) could move with constant velocity along the lattice, and a non-dissipative current is connected with this motion. This superconductivity mechanism was subsequently discussed in[2]. It was indicated in[3,4] that at finite but low temperatures ($T \ll \Delta$) the state of the system is close to the ground state [Eqs. (1) and (4)] and differs from the latter in the phase fluctuations (the "pseudogap model"). According to[3,4], a stationary superconducting current can flow in such a system with $n_s \propto n_0(\omega_0/\Delta)^2$ carriers (when $n_0$ is the number of carriers in the one-dimensional metal).

The main content in the present article is the statement that in a one-dimensional system with disorder there can be no motion of the structure, and an

arbitrarily small number of impurities at $T=0$ does not lead in any way to the appearance of small "friction"[3] but to identically zero conductivity. The physics of the situation is perfectly analogous to Mott localization[5] of the conduction electrons in a one-dimensional system[6,7]; in the present of defects, the eigenfunctions describing the vibrational degrees of freedom of the short-wave phonons ($k \sim 2k_F$) are localized in space. Therefore the deformations in the ground state of the system at $T=0$, which are determined by minimization of the elastic energy, do not correspond to the coherent structure (3).

In the absence of defects, the phase oscillations are described by the acoustic equation[3,4]:

$$\ddot{\phi}_1 - u^2 \phi_1'' = 0, \tag{5}$$

where $u^2 = d_1^2 v^2 / 4\pi v \Delta^2 \rho$. It will be convenient henceforth to use the dimensionless parameter $g_{ph}^2 = d_1^2/\pi v \rho \omega_0^2$ ($\rho$ is the mass density; then $u/v = g_{ph}(\omega_0/2\Delta)$.

The defects lead first of all to scattering of the conduction electrons and to localization of the electrons in the problem of one-dimensional conductivity.[6,7] For the sake of simplicity, we shall disregard the disorder introduced by the defect in the elastic matrix of the lattice. It is assumed that the effective radius of the potential of the "impurity" is small, shorter than the correlation length $\xi_0 = \hbar v/\Delta$, and that the impurity concentration is also small enough, i.e., the average distances $\bar{r}$ between the defects is larger than $\xi_0$.

In the initial approximation, on a segment between two impurities in the ground state, the equilibrium ion-displacement configuration with respect to which the oscillations take place retains the same form (3) as before. In the derivation of an equation to take the place of Eq. (4) for the low-lying phase oscillations, we simplify the analysis and retain only those impurity-potential components that transfer the electron, in the course of scattering, from one side of the "Fermi surface" to the other (say from $k_F$ to $-k_F$):

$$V^{\pm}(x-a) = |V| \exp[\pm i(2k_F(x-a) + \phi)]\delta(x-a). \tag{6}$$

The form of the potential (6) reflects the assumption of the "atomic" character of the defect.

It is necessary first to find new equilibrium positions of the oscillator

$$u^{\pm}(x) = |u^{\pm}(x)| \exp[\pm i(2k_F x + \phi(x)], \tag{3'}$$

where the coordinate dependence of $|u^{\pm}(x)|$ and $\phi(x)$ is due to the impurities, and then expand the total elastic energy in small deviations $u_1^{\pm}$ from these positions, up to quadratic terms.

The equilibrium condition is of the form ($T=0$):

$$\rho\omega_0^2 u^{\pm}(x) + 2d_1 \int G_{+-}^{(\omega)}(x,x)(d\omega/2\pi) = 0, \tag{7}$$

and the terms quadratic in $u_1^{\pm}$ in the total elastic energy are

$$\int dx\, \rho\omega_0^2 u_1^+(x) u_1^-(x) + 2d_1^2 \iint dx dx' \int (d\omega/2\pi)$$

$$\times \{u_1^+(x) G_{++}^{(\omega)}(x,x') G_{--}^{(\omega)}(x',x) u_1^-(x') + \frac{1}{2} u_1^+(x) G_{+-}^{(\omega)}(x,x') G_{+-}^{(\omega)}(x',x)$$

$$\times u_1^+(x')\}. \tag{8}$$

The exact electron Green's function in the absence of impurities, which enter in (7) and (8), make up a matrix whose form in terms of Fourier components is

$$\begin{pmatrix} G_{++}^{(\omega)} & G_{+-}^{(\omega)} \\ G_{-+}^{(\omega)} & G_{--}^{(\omega)} \end{pmatrix} = - \begin{pmatrix} i\omega + \xi & |\Delta| \exp i\phi \\ |\Delta| \exp -i\phi & i\omega - \xi \end{pmatrix} \frac{1}{\omega^2 + \xi^2 + \Delta^2} \tag{9}$$

[here $\xi = v(k \pm k_F)$; the factors $\exp(\pm 2ik_F x)$ in (7) and (8) can be left out]. Assuming Born impurities, it suffices to retain in (7) and (8) the increments linear in the potential $V$ (6). These increments yield in (7) corrections to (3): $u^{\pm}(x) = u_0^{\pm}(x) + u_V^{\pm}(x)$, where $u_V^{\pm}$ is proportional to $V$. The structure of the equation for $u_V^{\pm}$ is quite obvious:

$$\rho\omega_0^2 u_V^+(x) + 2d_1 \int dx' \int (d\omega/2\pi) \{ G_{++}^{(\omega)}(x-x') G_{--}^{(\omega)}(x-x') (d_1 u_V^+(x') + V^+(x')) + G_{+-}^{(\omega)}(x-x') G_{+-}^{(\omega)}(x'-x) (d_1 u_V^-(x') + V^-(x')) \} = 0. \tag{10}$$

In the part homogeneous in $u_V^{\pm}$, by virtue of the condition (4), appreciable cancellations take place. Using (8), we readily find that the integral kernels in (10) decrease exponentially over distances greater than $\xi_0$ from the impurities, i.e., they are equivalent to effective $\delta$ functions. A more detailed analysis shows that the kernel with the product $G_{++}^{(\omega)} G_{--}^{(\omega)}$ behaves at short distances $|x-x'| \ll \xi_0$ like $|x-x'|^{-1}$ [this behavior is the cause of the appearance of the logarithmic term in (4)], and therefore this term is the principal one, i.e., the amplitude of the effective potential for this term is larger by a factor $\ln 2E_F/\Delta \approx 1/g_{ph}^2$ than the same amplitude due to the term $G_{+-}^{(\omega)} G_{+-}^{(\omega)}$. It is also easy to verify that the impurity-induced perturbation $|u_V^{\pm}(x)|$ is concentrated in a region $\xi_0$ near the impurity:

$$d_1 |u_V^{\pm}(x)| = (V/g_{ph}^2) \cos(\phi + 2k_F a - \psi) \delta(x-a). \tag{11}$$

At the same time we have for the phase correction

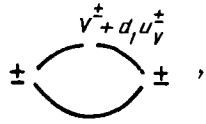
$$\phi_V'(x) = (4V\Delta/g_{ph}^2 v^2) \sin(\phi + 2k_F a - \psi) \delta(x-a). \tag{12}$$

On going through the impurity from left to right, the phase acquires an increment

$$\phi_V(x) = (4V\Delta/g_{ph}^2 v^2) \sin(\phi_a + 2k_F a - \psi) (x-a), \tag{12'}$$

i.e., the period of the superstructure (3) changes.

We proceed to calculate, with the same accuracy, the harmonic terms (8) in the energy. The terms linear in $V$ all have the schematic form




where different combinations (±) correspond to different sets of electron Green's functions (9). Because of the factor $1/g_{ph}^2$ in (11) and (12), it is possible to leave out $V^{\pm}$ from all these diagrams in comparison with $d_1 u_V^{\pm}$. The roles of (11) and (12') in these diagrams are different. Strictly speaking, $\phi_V$ in (12') is small over distances $x-a \gg \xi_0$ only if the parameter

$$\lambda = 2V/g_{ph}^2 v \tag{13}$$

is small. It was noted above that (12) has the meaning of the period of the superstructure. According to (9), the free term $G_{+-}^{(\omega)}G_{+-}^{(\omega)}$ in (8) contains a factor $\exp(2i\phi)$. It is easily verified that the contribution from the terms with $\phi_V$ means exactly the change of this factor on account of the change of the period of the structure.

After the calculations we obtain in lieu of (5) the following equation for the oscillations of the phase that is random over the impurity distributions:

$$\omega^2\phi_1(x) = -u^2\phi_1''(x) + (\omega_0^2 V/\Delta)\cos(\phi_a + 2k_F a - \psi)\,\delta(x-a)\phi_1(x). \tag{14}$$

We note that $\cos(\phi_a + 2k_F - \psi) > 0$. In the opposite case, the specifics of the one-dimensional problem would lead to solutions with $\omega^2 < 0$ near the given impurity.

From (1) and (12) follows the main conclusion: at small concentrations, the phases of all the impurities are "pinned": $\phi_a + 2k_F a - \psi = 2n\pi$. Otherwise the energy density (1) would acquire a term independent of the concentration $\sim\lambda^2\Delta^2/v$. It appears that at high temperatures a complete loss of phase coherence takes place. Comparing $\lambda^2\Delta^2/v$ with the contribution $\Sigma\omega n(\omega) \sim T/a$ of the thermal oscillations, we obtain for the corresponding temperature, $T^*$, the estimate:

$$T^* \sim \lambda^2\Delta^2/E_F. \tag{15}$$

The loss of phase coherence on an individual impurity takes place also at a finite concentration. Indeed, since $2k_F$ is not commensurate with the lattice period, the phases are always different at different at different impurities, and this leads inevitably to the appearance of $\nabla\phi \propto (\bar{r})^{-1}$. In light of the foregoing, however, this means apparently that the ground state of the system corresponds at a finite concentration to fixation of the phase on complicated "complexes" of impurity centers.

Let us return to Eq. (14), which has the same form as the Schrödinger equation with a repulsion potential. The coefficient of transmission through the barrier tends at small $\omega^2$ to zero[8]: $D \approx 4\omega^2/\lambda^2\omega_0^2 g_{ph}^2$. In this approximation, Eq. (15) reduces to the problem of motion between impenetrable barriers, and the probability density of a given distance $l$ between neighboring barriers is $(\bar{r})^{-1}\exp(-l/r)$. Inasmuch as in this case $\omega_n = nu/l$, we have as $\omega \to 0$ for the density of states

$$\rho(\omega)\,d\omega = \frac{u\pi}{\bar{r}\omega^2}\exp\left(-\frac{u\pi}{\bar{r}\omega}\right)d\omega\ .$$

In the same limit it is easy to determine the electromagnetic properties of the system. Generalizing the results of[4], we can write

$$\bar{j}(x) = -\frac{4\pi u^2}{v}\,\frac{\omega^2 e^2}{\pi}\int \overline{D_\omega^R(x, x')}\,A(\omega, x')\,dx',$$

where $\overline{D_\omega^R(x,x')}$ is the averaged retarded Green's function of Eq. (14):

$$D_\omega^R(x, x') = \sum_n \psi_n(x)\,\psi_n(x')[\omega^2 - \omega_n^2 + i\delta\omega]^{-1},$$

constructed on its eigenfunctions $\psi_n(x) = \sqrt{2/l}\,\sin(\pi nx/l)$. For the dielectric constant $\epsilon$ and the conductivity $\sigma(\omega)$ we obtain

$$\epsilon = \frac{8\pi}{3v} ce^2 \bar{r}^2; \qquad \sigma(\omega) = \frac{16u^3 c e^2}{v \bar{r} \omega^2} \exp\left(-\frac{u\pi}{\bar{r}\omega}\right)$$

We have thus shown that the conductivity vanishes at $T=0$. Whether a finite conductivity of the Fröhlich mode arises at $T \neq 0$ in the presence of defects is still unclear. At any low defect concentration, the temperature dependence of $\sigma(T)$ is of the semiconductor type as $T \to 0$. It is curious that the concentration does not enter in the estimate (15) of the temperature at which "compete breakaway" of the structure takes place.

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