Model of magnetic-field reconnection in a plane layer of collisionless plasma

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We consider the instability of a plane plasma layer placed in a magnetic field with rotating force lines. The instability constitutes excitation of the kinetic tearing mode. The overlap of the modes that develop near different resonant surfaces causes stochastic diffusion and reconnection of the magnetic-field force lines. The proposed mechanism may ensure reconnection of the interplanetary and geomagnetic fields at the boundary of the earth's magnetosphere.

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One of the essential shortcomings of extensively used theories of reconnection of the magnetic fields in collisionless plasma, based on the field dissipation as a result of the development of current instabilities near the vicinity of the neutral line, is the very small thickness of the magnetic-field inversion zone needed for the excitation of the plasma oscillations by the current (see, e.g., [1-3]). This, in particular, contradicts contemporary experimental data on the character of reconnection on the daytime side of the earth's magnetosphere. [4]

We propose below a method of reconnection in a plane layer of plasma with a magnetic field

$$\mathbf{B} = B_{oz}\,\mathrm{th}\,(x/\Delta)\mathbf{e}_z + B_{oy}\mathbf{e}_y\,; \quad b_y = B_{oy}/B_{oz} \tag{1}$$

and with a particle distribution function that depends only on two integrals of motion, namely the energy $mv^2/2$ and the y-component of the generalized momentum $p_y = mv_y + (e/c)A_{0y}$

$$f_{oj}(x, \mathbf{v}) = n_o(m_j/2\pi T_j)^{3/2}\exp\left[-\frac{m_j v^2}{2T_j} + \frac{u_j}{T_j}\left(m_j v_y + \frac{e_j}{c}A_{oy}\right) - \frac{m_j u_j^2}{2T_j}\right], \tag{2}$$

where n_0 is a constant and the drift velocity u_j is constant in a layer $u_j = (-2cT_j/e_jB_{0z})$. [5] The local particle density turns out to be inhomogeneous in this case

$$n(x) = n_o\,\mathrm{ch}^{-2}(x/\Delta). \tag{3}$$

The problem of instability of such a plane layer of plasma relative to oblique perturbations with a vector potential

$$\mathbf{A}(\mathbf{r}, t) = \mathbf{A}(x)\exp[-i\omega t + ik_y y + ik_z z] \tag{4}$$

reduces to a determination of the eigenvalue of an equation of the Schrödinger type[6,7]

$$\mathbf{A}''_{xx} - [k^2 + V_o(x) + \hat{V}_1(x, \omega, \mathbf{k})]\mathbf{A} = 0, \tag{5}$$

where $V_0(x) = -2\Delta^{-2}\cos^2\theta\cosh^{-2}(x/\Delta)$; θ is the angle between the wave vector and the c axis, $k^2 = k_y^2 + k_z^2$

$$\hat{V}_1(x, \omega, \mathbf{k})\,\mathbf{A} = \sum_j \frac{4\pi e_j^2}{c^2}\int d^3v\, f_{oj}\,\mathbf{v}[-i(\omega - k_y u_j)\int_{-\infty}^{0} d\tau\,(\mathbf{A}\mathbf{v})e^{-i\omega\tau + i\mathbf{k}\mathbf{r}(\tau)}].$$

We shall consider only the case of sufficiently large

$$b_y > \epsilon_e^{1/2}, \quad \epsilon_j = \rho_{jz}/\Delta << 1, \tag{6}$$

where $\rho_{jz} = \sqrt{2}cT_j^{1/2}m^{1/2}/|e_j|B_{0z}$ is the Larmor radius of the thermal particles in the magnetic field B_{0z}. In this case the drift approximation is applicable throughout, and the expression for $V_1(x)$ becomes

$$V_{1j}(x, \omega, \mathbf{k}) = 2\frac{\omega_{pj}^2}{c^2}\left(-\frac{\omega' - \omega_j^*}{k_\parallel v_{Tj}}\right)Z_2\left(\frac{\omega'}{k_\parallel v_{Tj}}\right), \tag{7}$$

where

$$\omega' = \omega - k_\parallel u_j \frac{B_{oy}}{B}; \qquad k_\parallel(x) = (\mathbf{k}\mathbf{B})/B$$

$$\omega_j^* = \frac{-\mathbf{k}c\,T_j[\nabla n\times\mathbf{B}]}{e_j n B^2}; \qquad Z_2(\xi) = \frac{1}{\sqrt{\pi}}\int_{-\infty}^{+\infty}\frac{Z^2 e^{-z^2}dz}{Z - \xi - i\epsilon\,\mathrm{sign}\,k_\parallel}, \ \epsilon\to 0. \tag{8}$$

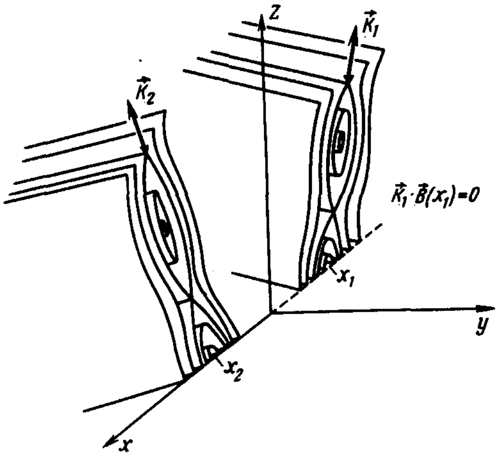

FIG. 1. Magnetic surfaces produced when two nonoverlapping modes with wave vectors K_1 and K_2 are excited in a plasma.

The buildup of the oblique oscillations is due to their resonant interaction with the particles near the singular surface $x=x_0$, defined by the equation $k_\parallel(x_0)=0$. The contribution of the vicinity of the singular region to Eq. (5) is described by the term $V_1(x)$ and can be regarded as a small perturbation. The equation for the shift of the energy level $E=-k_0^2=-\nu^2\Delta^{-2}$, $\nu=\frac{1}{2}[(1+8\cos^2\theta)^{1/2}-1]$ under the influence of the perturbation $V_1(x)$ is of the form[7]

$$B(1/2,\nu)(\nu^2-k^2\Delta^2)=\Delta\sum_j\int_{-\infty}^{+\infty}V_{1j}(x)\,\mathrm{ch}^{-2\nu}(x/\Delta)\,dx, \tag{9}$$

where $B(\frac{1}{2},\nu)$ is the β function.

This makes it possible to find the growth rate of the instability:

$$\omega=\omega_e^*(x_0)+i\,\frac{\epsilon_e^2\eta_e}{\sqrt{\pi}}\,\frac{kv_{Te}}{b_y}\,(\nu^2-k^2\Delta^2)^{1/2}B(1/2,\nu), \tag{10}$$

where $b_y=B_{0y}/B_{0z}$ and $\eta_e=(T_e+T_i)/T_e$. In a high-pressure plasma at $b_y<\sqrt{\epsilon_e}$, this expression goes over continuously into the well known result of Laval *et al.*,[8] and in the limit of very low pressure, at $b_y>(M_i/m_e)^{1/2}/\epsilon_e$, it joins smoothly the result obtained by Coppi.[9]

In the single-mode regime, development of the instability leads to formation of magnetic "islands" near the singular surface $x=x_0$ (see Fig. 1). As the perturbation amplitude B_{1x} increases, the thickness w of the magnetic islands increases:

$$w=\Delta\left(\frac{2B_{1x}}{k\Delta B_{0z}}\right)^{1/2}. \tag{11}$$

When calculating the contribution made to $V_1(x)$ by the particles moving over closed magnetic surfaces, i.e., inside the islets, account must be taken of the fact that $k_\parallel$ oscillates rapidly along the particle trajectory. One can therefore assume that $|\omega| \gg k_{\|} v_{\|}$ for such particles. Particles moving over unclosed magnetic surfaces contribute, as before, to the resonant interaction with the perturbations. If the island thickness is much larger than the dimensions of the region of interaction with the wave, i.e.,

$$w > |\omega| b_y \Delta / (k v_{Tj}) \lesssim \epsilon_j \Delta , \tag{12}$$

then the resonant interaction of the particles with the perturbation decreases rapidly. As a result, the dispersion equation (9) takes the form (for $\rho_{iz} > w > \rho_{ez}$).

$$\frac{1}{2} B\left(\frac{1}{2}, \nu\right) (\nu^2 - k^2 \Delta^2) \approx \left(\frac{\omega_{pe}}{c} \Delta\right)^2 (\omega' - \omega_e^*) \left[\frac{w}{\omega' \Delta} - i \pi^{1/2} \frac{b_y}{k v_{Te}} \frac{\rho_{ez}^2}{u^2}\right]$$
$$- i \pi^{1/2} \left(\frac{\omega_{pi}}{c} \Delta\right)^2 \frac{\omega' - \omega_i^*}{k v_{Ti}} b_y . \tag{13}$$

It follows therefore that in the nonlinear regime at $w > \rho_{ez}$ the growth rate of the instability in a diffuse plane layer, where $\epsilon_i < 1$, is somewhat larger than in the linear regime, and decrease slowly with increasing w.

With further increase of the amplitude of the considered oscillation mode, when $w > \rho_{iz}$, the growth rate of the instability begins to decrease very rapidly ($\sim w^{-4}$), although it does not vanish. Thus, just as in the hydrodynamic theory of tearing instability,[10] the development of magnetic islands following excitation of one oscillation mode can be stopped only by a quasilinear rearrangement of the plasma equilibrium.

In constrast to the plasma in toroidal systems of the tokamak type, the wave numbers of the perturbations in a plane infinite plasma layer are not quantized, so that resonant surfaces can occupy any position along the X axis. As a result it is always possible for the islets to overlap, and as a consequence, for "diffusion" of the magnetic force lines to take place. Following,[11] we can represent the diffusion coefficient of the magnetic field B, which in the case considered here $B_z = 8\pi n_0 (T_i + T_e)/B_{0z}^2 = 1$ coincides with the plasma diffusion coefficient in the form

$$D \sim v_{Ti}^2 \frac{B_{1x}^2}{B^2} \frac{1}{|k_{\|}(w)| v_{Ti}} . \tag{14}$$

Recognizing that the width of the "island" can be expressed simply in terms of the amplitude of the perturbation with the aid of Eq. (11) and that the resonant interaction of the ions with the perturbations decreases in inverse proportion to the cube of the islet dimensions at $w > \rho_{iz}$, we rewrite the expression for D in final form

$$D \sim v_A \Delta \epsilon_i^3 \left[\frac{\eta_e - 1}{\eta_e}\right]^{1/2} , \tag{15}$$

where $v_A \sim B_{0y}/\sqrt{4\pi n_0 M_i}$ is the Alfven velocity.

Substitution of this expression into the known formulas for the rates of inflow of the magnetic force lines into a plasma layer of length L_z (i.e., for the maximum "reconnection" rate) in the Parker-Sweet model[12] leads to the final result

$$U_x \lesssim v_A (\rho_{iz}/L_z)^{3/4} . \tag{16}$$

The thickness of the region in which the magnetic field expands its random walk turns out to be in this case much larger than the Larmor radius of the ions: $\Delta \sim L_z^{1/4}\rho_{iz}^{3/4}$.

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