Scattering of high-frequency sound near the λ transition in He

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It is shown that near the upper-transition point in He the damping of sound with frequency much larger than the frequency of the order-parameter relaxation is determined by scattering from quasistatic fluctuations of the transition parameter.

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The characteristic relaxation time of the order parameter in He near the λ point is $\tau_0=\xi/u_2$, where ξ is the correlation radius and u_2 is the second-sound velocity. As $T\to T_\lambda$, the value of τ_0 increases like ϵ^{-1}, where $\epsilon=(T-T_\lambda)/T_\lambda$. According to the generally accepted premises, the damping of first sound consists of a relaxation component,[1] which has a maximum at $\omega\tau_0=1$ below the

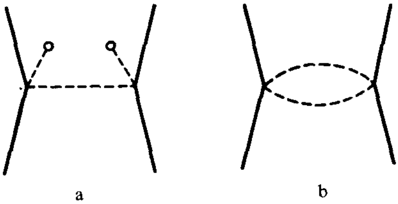

FIG. 1. Graphs describing the scattering of sound.

transition point and is equal to zero above this point, and a fluctuation component,[2] which is symmetrical with respect to the transition point. These theoretical premises agree well with experiment[3] in the low frequency region from 10^3 to 10^6 Hz.

However, Commins and Rudnick,[4] in experiments performed at a frequency 1 GHz, have observed in the region $\omega\tau_0 \gg 1$, a peak that seems to correspond to another relaxation time $\tau_1 \sim \xi/u_1$, where u_1 is the velocity of the first sound. Furthermore, the observed intensities, both of the peak and of the background, were much higher than the theoretical estimates based on the mechanisms considered above.

We propose in this paper a new mechanism of the damping of the sound wave in the frequency region $\omega\tau_0 \gg 1$, namely the scattering of the sound by quasistatic fluctuations of the order parameter. The fluctuations can be regarded as static by virtue of the condition $\omega\tau_0 \gg 1$.

We consider first the situation below the transition point. The scattering of the sound can be described by a known method,[5] by introducing the dependence of the sound velocity on the local value of the density ρ_s of the superfluid component. The fluctuation $\delta\rho_s$ is connected with the longitudinal fluctuation $\delta\eta$ of the order parameter by the relation $\delta\rho_s = 2\sqrt{\rho_s}\,\delta\eta$. The damping coefficient γ_1 is expressed in obvious fashion in terms of the correlator $\langle\delta\eta(\mathbf{q})\delta\eta(-\mathbf{q})\rangle$:

$$\gamma_1 = \frac{k^4}{4\pi}\frac{1}{u_1^4}\int d\theta\left(\frac{\partial u_1^2}{\partial\rho_s}\right)^2 \rho_s \,<\delta\eta(\mathbf{q})\delta\eta(-\mathbf{q})> . \tag{1}$$

Here $\mathbf{k}$ is the wave vector of the incident sound, θ is the scattering angle, and $q = 2k\sin(\theta/2)$. We use for the correlator *a* simplified representation[1)]:

$$<\delta\eta(\mathbf{q})\delta\eta(-\mathbf{q})> = \frac{m^2}{\hbar^2}\frac{T_\lambda\xi^2}{1+q^2\xi^2} . \tag{2}$$

The cross section of the scattering process described by formula (1) is represented by graph "a" of Fig. 1, where the solid, dashed, and dash-dot lines correspond to the sound, the order-parameter fluctuations, and to the condensate, respectively. In this graph we have replaced the vertex parts by their static values, which is correct in order of magnitude also at $q\xi \sim 1$. In the region $q\xi \gg 1$ the vertex parts and the Green's functions do not depend on the proximity to the transition point. Since $\rho_s \to 0$ as $T \to T_\lambda$, the contribution of the mechanism described by formula (1) vanishes at the transition point. This means that γ_1 has a maximum at $q\xi \sim 1$ [see (1) and (2)]. Since $k\xi = \omega\xi/u_1$, this maximum imitates a relaxation peak, with a characteristic time $\tau_1 \sim \xi/u_1$, although the mechanism of the phenomena is not connected with relaxation.

Let us estimate the order of magnitude of γ_1. It is necessary for this purpose to estimate $(\partial u_1^2/\partial\rho_s)_{\rho,T}$, the value of which near the transition point is

$$\frac{\partial u_1^2}{\partial\rho_s} = \rho\frac{\partial}{\partial\rho_s}\frac{\partial^2\phi}{\partial\rho^2},$$

where ϕ is the thermodynamic potential. When estimating the order of magnitude it suffices to separate from ϕ the term linear in ρ_s. It is known[6] that this term takes the form $A\epsilon^{4/3}\rho_s$, where A is a constant coefficient, $A\sim 2\times 10^8$ erg/g. Separating the most singular part, we obtain $(\partial u_1^2/\partial\rho_s)\sim A\epsilon^{-2/3}\rho T_\lambda^{-1}(\partial T_\lambda/\partial\rho)$. Near the transition point we have $\rho_s\approx 0.3\epsilon^{2/3}$ g/cm^3, $\xi\approx 3\times 10^{-8}\epsilon^{-2/3}$ cm, and $T_\lambda^{-1}(\partial T_\lambda/\partial\rho)\approx 2$ cm^3/g. Substituting these values in (1) and (2), we obtain an estimate for the absorption at the maximum, $\gamma_1\sim 10^2\times 10^3$ cm^{-1}. The large leeway in the value of γ_1 is due to the uncertainty in the values of the vertex parts and of the parameter A.

Besides the scattering described by graph "a" in Fig. 1., there is also the scattering described by graph "b," namely scattering by entropy fluctuations. This graph corresponds to the following contribution to the absorption coefficient:

$$\gamma_2 = \frac{k^4}{8\pi}\frac{1}{u_1^4}\int d\theta\int\frac{d^3p}{(2\pi)^3}\frac{T_\lambda^2}{((\mathbf{p}-q)^2+\epsilon)(\mathbf{p}^2+\epsilon)}\left(\frac{\partial u_1^2}{\partial\rho_s}\right)^2. \qquad (3)$$

The scales of γ_1 and γ_2 are the same. However, γ_2 contains in comparison with γ_1 an additional factor $\min[(\epsilon^{-\alpha}-1)/\alpha, \ln\epsilon]$, where α is the critical exponent of the heat capacity. γ_2 can therefore exceed γ_1 by one order of magnitude.

To verify the advanced arguments, one can use, besides the position of the peak, also the sharp frequency dependence of the value of the peak, $\sim\omega^3$, and also the fact that the scattering is elastic.

1)At small q we have $G\sim q^{-1}$,[6] but at $q\xi\sim 1$ the difference between the longitudinal and transverse correlators vanishes and the use of (2) is justified.

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