# Solitons in a system of parametrically excited waves

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It is shown that in a system of parametrically excited spin waves, stable nonlinear solitary waves (solitons) can be excited under certain conditions. Excitation of such solitons can lead to the appearance of spikes, which follow one another at strictly equal time intervals, on the plot of the energy absorbed by the system against the time.

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Under conditions of parametric excitation of spin waves, a situation can arise wherein the excitation threshold is minimal for one single pair of spin waves, with a certain wave vector $\mathbf{k}_0$. In this case there are excited in the medium packets that are narrow in $k$-space:

$$a(\mathbf{k}) = A(k - k_0)\exp(-i\omega_p t/2) \qquad (1)$$

($\omega_p$ is the frequency of the external high-frequency field of amplitude $h$).

The presence of a small parameter—the width of the packet—makes it possible to reduce the original dynamic equations for the classical amplitude $a(\mathbf{k})$ of the spin wave, by rewriting it in the language of the envelopes $A(x)$—the Fourier components of $A(\mathbf{k})$. Reduced equations of this type were first written down by L'vov and Rubenchik,[1] who investigated in detail different stationary spatially inhomogeneous solutions of these equations for the case when the homogeneous stationary solution is unstable.[1)] It turned out that only stationary spatially inhomogeneous solutions of the equations of the equations are unstable, with a growth rate larger than that of the homogeneous solution, so that the above-threshold state of a system of parametrically excited spin waves is essentially nonstationary. The spin-wave interaction parameters $T$ and $S$ of a number of substances (e.g., [3]) are such that the crystal state corresponding to excitation of a plane wave (homogeneous solutions) is stable. As will be shown below, in this case there can be excited in the medium plane spin waves with amplitudes that vary slowly in space (spatially inhomogeneous solutions). These solutions are stable, and their presence leads to interesting physical consequences. In particular, the unusual time dependence, observed in[3], of the power absorbed by a magnet can be explained by considering such inhomogeneous states along the crystal.

Confining ourselves to one-dimensional motions, we write down the reduced equation of motion for the complex envelope $A(x)$ [1]

$$i\left(\frac{\partial}{\partial t} + \gamma\right)A - hVA^* = -L^2\frac{\partial^2 A}{\partial x^2} + [\omega_{k_0} - \omega_{p/2} + T|A|^2 + 2S|A|^2]A \tag{2}$$

$\omega_{k_0}$ is the frequency of a spin wave with wave vector $\mathbf{k}_0$ determined by the dispersion law, $\gamma$ is the phenomenological damping, $L^2$ is the constant of the inhomogeneous exchange interaction, and $V$ is the interaction between the spin waves with wave vector $\mathbf{k}_0$ and the alternating field. The inhomogeneous equations (2) have two trivial solutions in the form

$$A = \pm a_0 \exp(i\Phi_0); \qquad \gamma/hV = \sin 2\Phi_0; \quad a_0 = \{(1 - \gamma^2/h^2V^2)/2S\}^{1/2}$$

$$\omega_{k_0} = \omega_{p/2} - Ta_0^2. \tag{3}$$

If $S > 0$ and $2S + T > 0$, then the homogeneous solution is stable with respect to perturbations $\alpha \sim \exp(\nu t + i\boldsymbol{\kappa}\cdot\mathbf{r})$.

Besides the homogeneous solution (3), Eq. (2) has also spatially inhomogeneous solutions of the type

$$A = a(x)\exp(i\Phi_0). \tag{4}$$

Stationary solutions from the class (4) can be periodic

$$a(x) = c(1 + c^2)^{-1/2}\,\mathrm{sn}\{xa_0(2S + T)^{1/2}L^{-1}(1 + c^2)^{-1/2},\ c\}, \tag{5}$$

where $c$ is the integration constant, the second constant is chosen such as to make $a(0) = 0$, and $sn(\cdots, c)$ is the elliptic sine with modulus $c$. The period of these solutions is $\tau = (4L/a_0)(1 + c^2)^{1/2}(2S + T)^{1/2}K(c)$, where $K(c)$ is the complete elliptic integral of the first kind.

If we choose the constant $c$ such that $\partial a/\partial x = 0$ at $|a| = |a_0|$, then we obtain the essentially nonperiodic solution

$$a(x) = -a_0\,\mathrm{th}(x/l), \qquad l = La_0^{-1}(2S + T)^{-1/2}. \tag{6}$$

The solution (6) goes over asymptotically as $|x| \to \infty$ into the homogeneous solution (3), and constitutes in fact a finite region of inhomogeneity in the system, thus analogous in its structure to an interphase boundary.

We note that this new state satisfies the same energy-bound relation $\gamma/hV = \sin 2\Phi_0$ as the homogeneous state (3). Consequently, in measurements of the absorption of energy by the crystal we can register only the instant of excitation of the spatially inhomogeneous state by observing the appearance, at a certain instant of time, of a spike whose area corresponds to the energy of the spatially inhomogeneous state

$$E = LS\,2^{1/2}(2S + T)^{1/2}(1 - \gamma^2/h^2V^2)^{-1/2}. \qquad (7)$$

If the system under consideration has some weak inhomogeneity, then the spatially inhomogeneous state of the crystal (6) turns out to be in an effective external field determined by the degree and the character of the inhomogeneity. The center of the transition layer begins to move along the crystal, and the structure of the packet changes negligibly to the extent that the inhomogeneity of the parameters of the system is small. In other words, we can seek a solution of Eq. (3) in the form

$$A = a(x - x_0(t))\exp(i\Phi_0) + \alpha(t, x), \qquad (8)$$

where $x_0$ is an integration constant, previously assumed equal to zero, but now dependent on the time.

Assuming for the sake of argument that the inhomogeneous parameter is the amplitude of the external high-frequency field $\tilde{h} = h(1 + \phi(x))$, we shall regard the derivatives of $\phi$ and $\alpha$ with respect to time to be of the same order of smallness. After linearization, we obtain

$$\hat{\mathcal{L}}^{(1)}_{\mu\nu} a^{\nu} = F_{\mu} \qquad (\mu, \nu = 1, 2), \qquad (9)$$

where

$$\hat{\mathcal{L}}^{(1)}_{11} = \hat{\mathcal{L}}^{(1)*}_{22} = -L^2\frac{\partial^2}{\partial x^2} - Ta_0^2 + 2(2S + T)a_0^2\,\mathrm{th}^2\left(\frac{x - x_0}{l}\right) - i\gamma,$$

$$\hat{\mathcal{L}}^{(1)}_{12} = \hat{\mathcal{L}}^{(1)*}_{21} = (2S + T)a_0^2\exp(i\Phi_0)\,\mathrm{th}^2\left(\frac{x - x_0}{l}\right) + hV, \qquad (10)$$

$$F_1 = F_2 = -i(a_0/l)\,\mathrm{ch}^{-2}\left(\frac{x - x_0}{l}\right)\exp(-i\Phi_0)\frac{\partial x_0}{\partial t} - hVa_0\exp(i\Phi_0)\,\phi(x)\,\mathrm{th}\left(\frac{x - x_0}{l}\right).$$

Knowing the solution of the homogeneous equation $\hat{\mathcal{L}}^{(1)*}\alpha^{(0)} = 0$, which is of the form

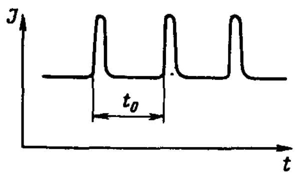
$$a^{(0)} = \mathrm{ch}^{-2}\frac{x - x_0}{l}\begin{pmatrix}\exp(-i\Phi_0)\\ \exp(i\Phi_0)\end{pmatrix} \qquad (11)$$

we easily obtain the equations of motion for the coordinates of center of the inhomogeneity of the investigated states, which is simply the condition under which the inhomogeneous equation (9) has a solution, with allowance for the slow variation of $\phi$:

$$\frac{\partial x_0}{\partial t} = -\left(\frac{3}{4}\right)\frac{h^2 V^2 l^2}{\gamma}\frac{\partial \phi}{\partial x} \tag{12}$$

Thus, the presence of some inhomogeneities in the system causes moving spatially inhomogeneous states—solitons—to be excited in the crystal.

The possibility of excitation of such moving inhomogeneity regions leads to the following physical picture. It is obvious that there should exist in the crystal a point corresponding to the center of that spatially inhomogeneous state which is most likely to be excited (this will most readily be simply the boundary of the crystal). The instant of excitation of the soliton, as indicated above, will be marked by a spike on the plot of the absorbed power against the time. The inhomogeneity region will then propagate along the crystal, and during that time the level of the absorbed power will be the same as for the homogeneous stationary state. After the lapse of a time $t_0 = D(\partial x_0/\partial t)^{-1}$ ($D$ is the corresponding dimension of the crystal), the inhomogeneity region will leave the sample and at that instant of time, a new region of inhomogeneity will become excited at the same point as the first region; the second region, in turn, will move through the sample. We should thus obtain in experiment absorbed-power spikes that follow one another at strictly equal time intervals, or the following picture:




which was observed in the experiments of Prozorova and Kotyuzhanskiĭ.[3]

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[1)]Here and henceforth the stability problem is taken to mean the investigation of the "internal"[2] stability of different states of the system of parametrically excited waves, in other words, stability with respect to perturbations of the amplitudes and phases of the already exciting spin waves.

[1]V. S. L'vov and A. M. Rubenchik, Preprint 1-72, Nucl. Phys. Inst., Siberian Div., USSR Academy of Sciences, 1972.
[2]V. E. Zakharov, V. S. L'vov, and S. S. Starobinets, Usp. Fiz. Nauk **114**, 609 (1974) [Sov. Phys. Usp. 17, 896 (1975)].
[3]A. L. Prozorova and B. Ya. Kotyuzhanskiĭ, Pis'ma Zh. Eksp. Teor. Fiz., this issue, p. 385.