Factorizability hypothesis in the quark model and large-angle elastic scattering of hadrons

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(Submitted March 15, 1977)
Pis'ma Zh. Eksp. Teor. Fiz. **25**, No. 9, 452–456 (5 May 1977)

The model of factoring quarks is supplemented by a dynamic hypothesis concerning the form of the scattering of a quark by a self-consistent potential. A simple formula, including a dependence on the number of the component quarks, is obtained for the hadron cross sections in the asymptotic form. A comparison is made with the experimental data on *pp* scattering.

PACS numbers: 12.40.Ce, 13.85.Dz

In the present paper we investigate the cross section of elastic hadron-hadron processes in the asymptotic region

$$s,\ t \to \infty, \qquad t/s \ \text{is fixed,} \tag{1}$$

by using the model of factorized quarks (MFQ).[1] In this model we assume the following picture of the interacting hadrons. It is proposed that when the hadrons collide, the quarks making up the hadrons produce a self-consistent field V_{eff}, in which they are scattered in independent fashion. For independent events, the probability of emission of an entire combination of quarks through an angle θ is equal to the product of the individual probabilities of the scattering of each quark through the angle θ. We consequently assume for the amplitude of the scattering or two hadrons A and B

$$M_{AB}(\theta) = \Sigma \prod_{i=1}^{n} g_i(\theta) \prod_{j=1}^{m} g_j(\theta), \tag{2}$$

where n and m are the numbers of quarks in the hadrons A and B, respectively, $g_i(\theta)$ is the amplitude for the scattering of the ith quark by the self-consistent potential V_{eff}, and the summation is carried out over all the possible processes with exchange of identical quarks.[1,2]

We supplement the model of factorizing quarks[1] by the dynamic assumption concerning the explicit form of the potential V_{eff}, specifying for it an expression in the relativistic configuration representation (RCR) first introduced in[3]. According to[3], the transition from the momentum representation in the RCR is effected not with the aid of a Fourier-Bessel transformation, but with the aid of expansions on the Lorentz group. The role of the plane waves $\exp(i\mathbf{p}\cdot\mathbf{r})$ is assumed here by the functions[4] (using the notation of[3] and $\hbar = c = 1$)

$$\xi(\mathbf{p}, \mathbf{r}) = \left(\frac{p_0 - \mathbf{p}\mathbf{n}}{M}\right)^{-1 - irM}$$

which realize the unitary (infinite-dimensional) irreducible representations of the Lorentz group. As a result, the amplitude for the scattering of a quark by a spherically-symmetrical potential $V_{eff}(r)$ is given in the Born approximation by the expression[3]

$$g_i(\theta) = 4\pi \int_0^\infty \frac{\sin rM_q\gamma_i}{rM_q\gamma_i} V_{eff}(r) r^2 dr, \tag{3}$$

where $y_i = \cosh^{-1}(1 - t_i/2M_q^2)$ is the rapidity conjugate to the momentum transfer t_i per quark ($t_i \approx t/n^2$), and M_q is the quark mass and serves as a parameter.

We now choose $V_{eff}(r)$ in the RCR in the form

$$V_{eff}(r) \sim \frac{1}{4\pi r^2}\delta(r). \tag{4}$$

Substitution of (4) in (3) yields[5]

$$g_i(\theta) \sim \frac{\gamma_i}{\operatorname{sh}\gamma_i} = \frac{2M_q^2 \ln\left(1 - \frac{t_i}{2M_q^2} + \frac{1}{2M_q^2}\sqrt{t_i(t_i - 4M_q^2)}\right)}{\sqrt{t_i(t_i - 4M_q^2)}}. \tag{5}$$

It is easily seen that at $t_i \ll 4M_q^2$ we have $y_i/\sinh y_i \approx 1$, and at large momentum transfer $t_i \gg 4M_q^2$ we have[6]

$$\frac{\gamma_i}{\operatorname{sh}\gamma_i} \approx 2M_q^2 \frac{\ln(|t_i|/M_q^2)}{|t_i|}. \tag{6}$$

In the kinematic region (1), where t/s is fixed, we obtain after substituting (5) in (2) the amplitude of the elastic scattering of hadron A by B

$$M_{AB}(\theta) \sim \prod_{i=1}^{n} \frac{\gamma_i}{\operatorname{sh}\gamma_i} \prod_{j=1}^{m} \frac{\gamma_j}{\operatorname{sh}\gamma_j}. \tag{7}$$

For pp scattering through 90° in the cms, i.e., $t = -s/2$, we obtain from (7), in particular, the formula

$$\frac{d\sigma}{dt}(pp \to pp) \sim \frac{1}{s^2}\left[\frac{\ln s/18M_q^2}{s/18M_q^2}\right]^{12}, \tag{8}$$

which can be represented in the customary power-law form

$$\frac{d\sigma}{dt}(pp \to pp) \sim (s/18M_q^2)^{-n_{\rm eff}(s,\,\theta = 90^\circ)}, \tag{9}$$

where, however,

$$n_{\rm eff} = 14 - 12\frac{\ln(\ln s/18M_q^2)}{\ln s/18M_q^2} \tag{10}$$

increases with increasing s, in agreement with the known experimental data on pp scattering through 90°.[7] $n_{\rm eff}$ depends on s in a manner similar to (10) also in theories with asymptotic freedom.[8]

The results of the reduction of the experimental data on elastic pp scattering through various angles by means of formulas (7) and (8) of our dynamic model of factorizing quarks (FDMQ) are listed in Table I, which gives for comparison also the values of χ^2 per degree of freedom, χ^2_{df}, which are obtained when the reduction is by the formulas of the quark counting[9] $d\sigma/dt \sim s^{-10}$. The effective mass of the quark obtained in the reduction is $M_q \approx 0.2$ GeV.

If formula (7) is extended to include the case of quark-quark scattering, then we obtain from (7)

$$\frac{d\sigma}{dt}(qq \to qq) \sim \frac{1}{s^2}\left(\frac{y_q}{\operatorname{sh} y_q}\right)^4 \approx \frac{1}{s^2}\,\frac{[2M_q^2 \ln|t|/M_q]^4}{t^4}, \tag{11}$$

whereas the quark counting[9] yields $d\sigma/dt\,(qq \to qq) \sim s^{-2}$. It was shown recently in[10], however, that the experimental data on inclusive processes can be adequately described only by assuming for the quark-quark scattering a purely phenomenological relation of the type $d\sigma/dt(qq \to qq) = A/s^2t^2$. This

TABLE I.

θ, deg	s, GeV²	$-t$, GeV²	χ^2_{df}	
			DMFQ	$d\sigma/dt \sim s^{-10}$
38	36—61	3.5—6.1	2.52	4.49
68	19—52	5.0—15	1.90	8.93
75	19—49	6.0—14	3.11	9.12
90	24—43	10.0—20	1.48	2.61

parametrization was assumed in[10] for the description of the data in the interval $s \approx 10-20$ GeV^2, where the fraction of energy per quark is $s_q \approx 1-2$ GeV^2. In this region, the logarithmic terms contained in our formula (11) make an appreciable contribution, and formula (11) agrees with the phenomenological parametrization of the quark-quark cross section obtained in[10].

We plan in the future, in analogy with[11], to take into account the angular dependence that results from the presence of the spin variables, and to use our model for a description of inclusive reactions in meson-baryon elastic processes.

The authors thank D. I. Blokhintsev, V. G. Kadyshevskiĭ, P. S. Isaev, V. A. Meshcheryakov, and A. T. Filippov for stimulating discussions.

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