Are additional heavy quarks necessary to explain the observed behavior of $R(s)$?

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The sum rules obtained by Krasnikov and Chetyrkin (1977) at finite energies for $R(s) = \sigma(e^-e^+\text{hadrons}) / \sigma(e^-e^+ \to \mu^-\mu^+)$ are compared with experiment. It is found that the four-quark model agrees satisfactorily with the experimental data (with the heavy lepton taken into account.). It is shown that the five-quark model (with $Q_5 = -1/3$) describes the experimental results in the best manner.

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It is quite possible that strong interactions of quarks are described by the asymptotically free gauge field theory (AFGFT) with the color SU(3) group as the gauge. Since, however, there is no clear understanding of the mechanism that "retains" the quarks inside the hadrons, the AFGFT cannot be used directly for calculations of physical processes on the mass shell even at high energies.[2] For the process $e^-e^+ \to$ hadrons, for example, we have only implicit information, namely, the asymptotic behavior of the quantity[3–5]

$$T(s) = \int_{4m_\pi^2}^{\infty} \frac{R(s')ds'}{(s+s')^2} \underset{s\to\infty}{=} \frac{a}{s} + \frac{b}{s\ln(s/\Lambda^2)} + o\left(\frac{1}{s\ln s}\right), \tag{1}$$

where $a = 3\sum_1^n Q_i^2$, $b = 12n/(33-2n)$, Q_i is the charge of the ith quark, and n is the number of valence quarks.

The problem of an experimental verification of Eq. (1) (and by the same token the problem of obtaining information on the fundamental quantities n and Q_i) comes up, on the one hand, against the fact that $R(s)$ is known only on a finite interval of s [if we verify Eq. (1) directly, as is done in[6]], and against the instability of the operation of analytic continuation on the other hand [if we use the analytic continuation of the asymptotic form of $T(s)$ to obtain the analytic form of $R(s)$]

It was shown in[1], however, that the presence in $T(s)$ of a leading asymptotic form of the type a/s is equivalent to validity of the following sum rule at finite energies (SRFE)

$$\langle R\rangle(s) \underset{s\to\infty}{=} a + o(1), \quad \langle R\rangle(s) \equiv s^{-1}\int_{4m_\pi^2}^{s} R(s')ds'. \tag{2}$$

Under the additional assumption that the difference $R(s) - a$ does not reverse sign starting with a certain value of s, we can obtain also the next term in the asymptotic expansion of the quantity $\langle R\rangle(s)$

$$\langle R\rangle(s) \underset{s\to\infty}{=} a + b/\ln(s/\Lambda^2) + o(\ln^{-1}(s)). \tag{3}$$

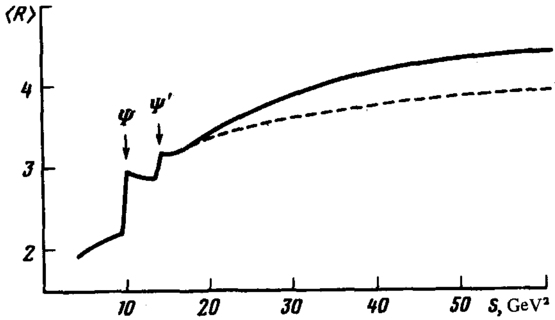

FIG. 1. Solid curve—plot of the function $\langle R_{\text{expt}}\rangle$ (s); dashed curve—plot of the function $\langle R^h_{\text{expt}}\rangle$ (s).

We note that the SRFE (3) is equivalent (under the aforementioned additional assumption) to Eq. (1) and being by the same token independent of any concrete dynamic assumptions concerning the mechanism whereby the quarks are retained, constitutes simply a formulation that makes Eq. (1) convenient for comparison with experiment.

For a comparison of the SRFE (3) with the experimental data, we show in Fig. 1 a plot of the quantity $\langle R^h_{\text{expt}}\rangle$ obtained by subtracting from the experimental value of $R(s)$[7,8] the contribution due to the existence of a heavy charged lepton[9] L with a mass $m_L = 2$ GeV/c^2

$$R_L(s) = B_L\left(1 + \frac{2m_L^2}{s}\right)\sqrt{1 - \frac{4m_L^2}{s}}$$

where we have put $B_L = 0.8$.[10] If we assume that in the region $s \geqslant 50$ GeV^2 the correction terms to the SRFE (3) are negligibly small [This assumption is favored by the fact that the $\langle R^h_{\text{expt}}\rangle(s)$ flattens out $s \geqslant 40$ GeV^2, and also by the fact that even terms of the form $\ln^{-1}(s)$ $(\leqslant 0.3$, see below) are already relatively small], then Fig. 1 yields an upper bound of the sum of the squares of the quark charges

$$a \lesssim \langle R^h_{\text{expt}}\rangle\ (s \approx 60\ \text{GeV}^2) \approx 4.0\,. \tag{4}$$

It is obvious that discarding the $\sigma[\ln^{-1}(s)]$ terms in (3) at $s = 50$—60 GeV^2 means neglecting the quark masses in this region, a permissible procedure only if the new heavy quarks are not significantly heavier than the c-quark (or, to the contrary, their mass is very large, in which case they are not contained in the sum over i). The inequality (4) is satisfied by three models: A—(standard) $n=4$, $Q_4=\frac{2}{3}$; B—$n=5$, $Q_4=\frac{2}{3}$, $Q_5=-\frac{1}{3}$; C—$n=6$, $Q_4=\frac{2}{3}$, $Q_5=Q_6=-\frac{1}{3}$. By fitting the value of $\langle R^h_{\text{expt}}\rangle(60)$ to formula (3) with allowance for the restriction $0.21 \leqslant \Lambda \leqslant 0.7$ GeV obtained for Λ on the basis of an analysis of the experimental data on elastic e–p scattering and on the hadron widths of the particles[2,11] we obtain for the models in question

$$\langle R_A \rangle (60) = 3.7; \quad \langle R_B \rangle (60) = 4.0; \quad \langle R_C \rangle = 4.3;$$

$$\Lambda_A = 0.7 \text{ GeV}; \quad \Lambda_B = 0.5 \text{ GeV}; \quad \Lambda_C = 0.2 \text{ GeV}.$$

Since the cited systematical error in the experimental data on $R(s)$ amounts to $\sim 10\%$ in the entire interval of variation of s, we find that all three models describe satisfactorily the experimental data, the best agreement being observed for the five-quark model with $Q_5 = -\frac{1}{3}$. [12]

We note that similar conclusions were reached also in[2], where a somewhat different averaging was used:

$$\bar{R}(s, \Delta) = \frac{\Delta}{\pi} \int_{4m_\pi^2}^{\infty} \frac{R(s')ds'}{(s - s')^2 + \Delta^2}. \quad (5)$$

They have shown, using additional dynamic assumptions, that the AFGFT make it possible to calculate $\bar{R}(s)$ reliably if Δ is suitably chosen. The averaging in (5), however, is not over a finite energy, as a result of which the comparison of the theory with experiment was carried out in the energy interval $8 \leq S \leq 38$ GeV^2.

Thus, the methods used in[2] and in the present paper are mutually complementary, and the agreement of the results implies indirectly compatibility of both approaches.

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