Polarization loss and induced electric charge of neutrinos in plasmas

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Neutrinos interact with an electromagnetic field and with charged particles in a plasma by virtue of the electromagnetic structure which arises from electroweak interactions [S. Weinberg, Phys. Rev. Lett. **19**, 1264 (1967); A. Salam, in Elementary Particle Physics, Almqvist and Wiksells, Stockholm, 1968, p. 367] and which is determined by the electromagnetic form factor $F_\nu(q^2)$.

In a vacuum, the normalization $F_\nu^{\rm vac}(0) = 0$ corresponds to a strictly zero electric charge for neutrinos. This property should prevail in a perturbation theory of any order; it has been tested in the electroweak theory[1] at the level of single-loop radiation corrections.[4]

In a dispersive medium, e.g., an isotropic plasma, the interaction of neutrinos with the macroscopic self-consistent electromagnetic field can be described by the electric form factor[1)]

$$F_\nu^{med}(\omega, k) = \frac{G_F(1 + 4\sin^2\theta_W)q^2}{4\pi\sqrt{2}\,\alpha}\,(\epsilon_l(\omega, k) - 1)\,, \tag{1}$$

where the longitudinal dielectric constant

$$\epsilon_l(\omega, k) = 1 - \Pi_{00}(\omega, k)/k^2 \tag{2}$$

is determined by the component Π_{00} of the polarization tensor or the medium and of the vacuum, $\Pi_{\mu\nu}(\omega,k)$ (Ref. 5). In (1), G_F is the Fermi weak-interaction constant (M_Z is the mass of the Z boson), θ_W is the Weinberg angle,[1] and e is the charge of an electron ($e^2 = 4\pi\alpha$, $\alpha = 137^{-1}$). The form factor in (1) is substantially larger than the vacuum form factor for a given momentum transfer q^2 (by a factor of about $137 = \alpha^{-1}$).

At small momentum transfer, $|q^2|/M_{Z,W}^2 \ll 1$, it is a sufficient approximation to consider only the contributions of leptonic loops to the polarization of the medium and vacuum. The ions, which are assumed to be immobile, keep the homogeneous system electrically neutral.

In a plasma with a Debye length r_D, in contrast with a vacuum, the form factor in (1) has the normalization

$$\lim_{k\to 0} F_\nu^{med}(0, k) = -\frac{G_F(1 + 4\sin^2\theta_W)}{4\pi\sqrt{2}\,\alpha r_D^2} \equiv \frac{e_\nu^{\rm ind}}{e}, \tag{3}$$

which determines a nonvanishing electric charge of the neutrinos, $e_\nu^{\rm ind}$ (in a vacuum, $r_D \to \infty$). The charge $e_\nu^{\rm ind}$ results from the weak attraction of the electrons of the medium to the neutrinos.

The overall system is electrically neutral; i.e., global gauge invariance is not disrupted by a repulsion of charges of opposite sign (positrons) or by the induction at the boundary of the medium (at infinity in the case of an unbounded medium) of an exactly equal neutralizing charge (of ions) distributed isotropically along the boundary of the surface (a "capacitor").

The induced electric charge of neutrinos was derived in Refs. 2 and 3 in a treatment ignoring charged currents. A new dependence on the Weinberg angle θ_W appeared in (3) when the complete Born amplitude for low-energy νe scattering was used[6]:

$$M_{Z+W} \sim \sqrt{2}\, G_F \left(\bar{\nu}\gamma_\mu \frac{(1-\gamma_5)}{2}\nu\right)\left(\bar{e}\gamma^\mu \frac{(1+4\sin^2\theta_W-\gamma_5)}{2}\, e\right) \quad 6$$

In this matrix element, the source of parity-conserving electromagnetic excitations of the medium could be only the vector part of the weak current of electrons, which is proportional to $\sim(1+4\sin^2\theta_W)$. As a result, the charge in (3) does not vanish, regardless of the angle θ_W; the result would only intensify the effects predicted in Refs. 2 and 3.

Let us examine the matrix element for the polarization emission of longitudinal plasmons[2)] by a massless neutrino,

$$\langle p'q\,|S|\,p\rangle = -i(2\pi)^4\left(4\epsilon_{p'}\epsilon_p\,|\,q^2\,\frac{\partial\,{\rm Re}\,\epsilon_l}{\partial\omega}\,|\right)^{1/2}\delta^{(4)}(p'+q-p)J_\mu^{(\nu)}e^\mu\,, \qquad (4)$$

corresponding to the diagram in Fig. 1.

Here the conserved electromagnetic current of neutrinos ($J_\mu^{(\nu)}q^\mu = 0$) can be written as follows, where we are taking into account the definition of the longitudinal polarization tensor in statistical[5] QED, $\Pi_{\mu\nu}^{(l)} = -q^2[\epsilon_l(\omega,k)-1]e_\mu e_\nu/4\pi$, and of the unit ($e^2=-1$) polarization 4-vector of a longitudinal plasmon, $e_\mu = (k,\omega\hat{\mathbf{k}})/\sqrt{q^2}$:

$$J_\mu^{(\nu)} = e F_{med}^{(\nu)}(\omega,k)\bar{\nu}(p')\gamma_\mu \frac{(1-\gamma_5)}{2}\nu(p)\,, \qquad (5)$$

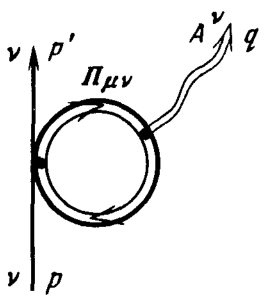

FIG. 1. Feynman diagram of a polarized emission of neutrinos. The double loop represents the polarization tensor $\Pi_{\mu\nu}$ in the medium.

where the form factor $F^{(\nu)}_{\rm med}(\omega,k)$ automatically takes the form in (1).

Form factor (1) was derived by an independent method in Ref. 7, where a quantum kinetic equation was derived systematically, by the Bogolyubov method, for neutrinos with allowance for the self-consistent field, determined by the amplitude for νe scattering.

The polarization energy loss of a massless ($\epsilon_p = p$) neutrino found with the help of (4) reduces to the single integral

$$\frac{dW^{(\nu)}_{polar}}{dl} = \frac{G_F^2(1+4\sin^2\theta_W)^2}{\pi e^2}\int_{k_{min}}^{k_{max}} \frac{\omega_j(k) q_j^4\, dk}{k\left|\frac{\partial \mathrm{Re}\,\epsilon_l}{\partial\omega}\right|_{\omega=\omega_j}}\left[1-\frac{\omega_j(k)}{p}+\frac{q_j^2}{4p^2}\right], \quad (6)$$

where the longitudinal-plasmon emission frequencies [the solutions of the dispersion relation $\epsilon_l(\omega_j,k) = 0$] satisfy energy conservation, $\omega_j = p - |\mathbf{p}-\mathbf{k}|$, which reduces in the semiclassical approximation ($k \ll p$) to the (Vavilov-) Čerenkov condition $\omega = \mathbf{kc}$. For a Langmuir plasmon ($\omega_j \approx \omega_{pe}$) the lower limit is $k_{\min} = \omega_{pe}$, while the upper limit (for relatively hard neutrinos, $p > r_D^{-1}$) is $k_{\max} \approx r_D^{-1}$. In this case we find the neutrino loss from (6):

$$\frac{dW^{(\nu)}_{polar}}{dl} \approx \frac{G_F^2(1+4\sin^2\theta_W)^2\omega_{pe}^2}{2\alpha r_D^4}, \quad \left(\omega_{pe} = \sqrt{\frac{4\pi\alpha n_0}{m_e}},\ n_0 = \frac{p_F^3}{3\pi^2}\right). \quad (7)$$

The numerical value of (7) for metals is $\sim 10^{-25}$ eV/cm.

Let us compare the collective loss in (7) with the collisional loss for soft neutrinos in a degenerate, isotropic, metal plasma ($p_{F_e} \ll m_e$), given by

$$\frac{dW^{\nu}_{coll}}{dl} \cong \frac{3G_F^2}{8\pi^2}\frac{\omega_{pe}^2}{\alpha}\frac{p_{F_e}^2\epsilon_\nu^3}{m_e}, \quad r_D^{-1} < \epsilon_\nu \ll m_e, \quad (8)$$

The ratio of the losses in (7) and (8) is

$$\frac{dW^{\nu}_{polar}/dl}{dW^{\nu}_{coll}/dl} \sim \alpha^2 (m_e/\epsilon_\nu)^3.$$

For neutrinos from a tritium source ($\epsilon_\nu \sim 1$ keV), the polarization loss is two orders of magnitude greater than the collisional loss, indicating the importance of collective mechanisms in the interactions of neutrinos with media (the particular mechanism discussed here is not the only one possible). For hard neutrinos ($\epsilon_\nu \gg m_e$) the ratio of losses tilts in favor of the collisional loss [in a metal, we would have $10\alpha^2(m_e/\epsilon_\nu)^2 \ll 1$].

The collisional energy loss of neutrinos, which plays an important role in the collapse dynamics of stars, is substantially lower than the loss due to electromagnetic emission (in the energy range $\omega_{pe} \ll \epsilon_\nu \ll p_{F_e}$).

[1)]Here we are using a system of units of $\hbar = c = 1$; the Feynman metric, $q^2 = q_\mu q^\mu = \omega^2 - \mathbf{k}^2$; $\mu = 0, 1, 2, 3$; and the standard representation of the Dirac γ matrices, with $\gamma_5 = \gamma_5^+ = i\gamma_0\gamma_1\gamma_2\gamma_3$.

[2)]For photons in a medium (of plasmons), the "mass" q^2 is nonzero ($q^2 \neq 0$).

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