A scalar meson is a dilaton in QCD

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A scalar meson/dilaton appears in QCD in a conformal bosonization of the generating functional of QCD for quark currents. The dynamics of π and σ mesons is described by an effective Lagrangian like that in the linear sigma model. The mass of the sigma meson is determined substantially by a gluon condensate.

A scalar isosinglet meson σ was originally introduced in the linear sigma model.[1] The phenomenological description of this meson has recently been discussed extensively in the literature (Ref. 2 and the bibliography there). In the nonlinear sigma model, the σ meson is eliminated as an independent degree of freedom from the theory. It can be seen that the phenomenological description of the σ meson is unsatisfac-

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tory because the field $\sigma(x)$ is related to only a breaking of chiral symmetry and a quark condensate. In quantum chromodynamics (QCD), low-energy processes are determined by both a quark condensate and a gluon condensate. In the present letter we show that a scalar meson appears in QCD in a natural way as the result of a breaking of a conformal symmetry upon the formation of a gluon condensate. We examine the joint dynamics of π and σ mesons in the low-energy region of QCD. This dynamics is described by an effective Lagrangian which is derived in QCD by a combined chiral plus conformal bosonization of the quark degrees of freedom in the spirit of Ref. 3.

We consider the quark Lagrangian $\mathscr{L}_\psi = \bar\psi \not{D} \psi$, where the complete Dirac operator $\not{D}(V,A,S,P,G)$ depends on the external vector, axial, scalar, and pseudoscalar isospin fields and also on the gluon field G_μ. The mass of the quarks is incorporated in S. In contrast with Ref. 3, we use not a chiral transformation but a chiral plus conformal transformation of the fields, $V_\mu \Rightarrow V_\mu^\Phi,\ldots$, which is ultimately equivalent to the transformation

$$\not{D}(V,\ldots) \Rightarrow \not{D}(V^\Phi,\ldots) = \Phi \not{D}(V,\ldots)\Phi,$$

where $\Phi = \exp(\sigma + i\gamma_5 \Pi)/2F_\pi$, and $\sigma(x)$ and $\Pi(x)$ are the fields of the σ and π mesons, and F_π is the decay constant of the π meson. The calculation of the effective Lagrangian follows and generalizes the method developed previously[3] for the case $\sigma = 0$. The effective action $W_{\rm eff}(\pi,\sigma)$ is given by the expression

$$W_{\rm eff}(\pi,\sigma) = -i \ln\{Z_\psi(\not{D}) Z_\psi^{-1}(\Phi \not{D} \Phi)\},$$

where $Z_\psi(\not{D})$ is the generating functional for quark currents in the low-energy region, L:

$$Z_\psi(\not{D}) = \int_L D\bar\psi D\psi \exp i \int \bar\psi \not{D} \psi d^4x .$$

Region L is determined by quark and gluon condensates.[3]

The effective Lagrangian for the fields $\sigma(x)$ and $\Pi(x)$ depends on only their combination $\varphi = F_\pi U \exp(-\sigma)$, where $U = -\exp(i\Pi/F_\pi)$ is the chiral field. Here is an expression for $\mathscr{L}_{\rm eff}$ in the case in which there are no external fields ($V_\mu = A_\mu = P = 0$, $S = m_q \Pi_f$, where m_q is the mass of a quark, and Π_f is a unit matrix in isospin space):

$$\mathscr{L}_{\rm eff}(\varphi) = \frac{1}{4}\,{\rm tr}\,(\partial_\mu \varphi^+ \partial^+ \varphi) - \mathcal{V}(\varphi) + \mathcal{L}^{(4)}(\varphi) .$$

$$\mathcal{V}(\varphi) = {\rm tr}\left\{ \frac{\tilde{C}_g}{48F_\pi^4}(\varphi^+\varphi)^2 - \frac{C_g}{24}\ln\frac{\varphi^+\varphi}{F_\pi^2} \right\} + (m_\pi^2/2F_\pi)\,{\rm tr}\,\varphi^+\varphi(\varphi+\varphi^+), \quad (1)$$

$$C_g = \tilde{C}_g - 18 m_\pi^2 F_\pi^2 = \langle \frac{\alpha_s}{\pi} \sum_a (G^a_{\mu\nu})^2 \rangle ,$$

The term $\mathscr{L}^{(4)}(\varphi)$ contains contributions from four derivatives of the fields, of which the most important for an understanding of the physics of π and σ mesons are (a) the term which generates a tachyon,

$$\mathcal{L}_T^{(4)} = \frac{\text{tr}}{320\pi^2}\left\{ 8\frac{\partial^2\varphi\partial^2\varphi^+}{\varphi\varphi^+} - (\varphi^{-1}\partial^2\varphi)^2 - (\partial^2\varphi^+(\varphi^+)^{-1})^2 \right\},$$

and (b) the vertices which are responsible for the decay $\sigma\to\pi\pi$,

$$\mathcal{L}_{\sigma\pi\pi}^{(4)} = -\frac{\text{tr}}{32\pi^2}\frac{\partial_\mu\varphi^+\partial_\nu\varphi}{\varphi^+\varphi}\{\varphi^{-1}\partial^\mu\partial^\nu\varphi + \partial^\mu\partial^\nu\varphi^+(\varphi^+)^{-1}\},$$

where the mass of the π meson is expressed in terms of the quark condensate, $F_\pi^2 m_\pi = -m_q\langle\bar{\psi}\psi\rangle$. The first two terms in the potential $\nu(\varphi)$ are chiral-invariant and have a minimum at $\varphi = F_\pi$; the last term breaks the chiral symmetry. The fluctuations of the field $\sigma(x)$ near the minimum of $\varphi = F_\pi$ are described by the dilaton mass

$$\bar{m}_\sigma^2 = \frac{2}{3}\frac{C_g}{F_\pi^2} + 3m_\pi^2, \tag{2}$$

which is determined primarily by the gluon condensate, C_g. The effective mass in (2) is meaningful at energies not exceeding 300 MeV. In this region, the linear sigma model in (1) reduces a nonlinear model which describes exclusively π mesons. The dilaton mass is large: $\bar{m}_\sigma = 0.8 - 1.5$ GeV for the gluon condensate in the range $C_g = (300\text{–}400\ \text{MeV})^4$. At these low energies, the kinetic term in the Lagrangian for the field $\sigma(x)$ is normalized to $F_\sigma = F_\pi$; this approach involves the chiral symmetry of the complete kinetic term for the fields $\sigma(x)$ and $\Pi(x)$ in the region of soft mesons.

The term $\mathscr{L}_T^{(4)}$ describes tachyons in both the π and σ channels. The tachyons appear because of a cancellation of the total number of quark degrees of freedom in (1) to fields $\Pi(x)$ and $\sigma(x)$. The position of a tachyon characterizes the applicability of the effective description in (1) in the corresponding channel. By virtue of the tachyon term $\mathscr{L}_T^{(4)}$, both the mass of the dilaton and the normalization of the field F_σ depend on the energy. On the mass shell we have

$$m_\sigma^2 = \frac{20\pi^2F_\pi^2}{3}\left(\sqrt{1+\frac{3\bar{m}_\sigma^2}{10\pi^2F_\pi^2}} - 1\right), \qquad F_\sigma^2 = \frac{3m_\sigma^2}{20\pi^2} + F_\pi^2. \tag{3}$$

At the mass $m_\sigma = 800$ MeV we have $F_\sigma \cong 1.5F_\pi$, indicating a breaking of chiral symmetry of the kinetic term in this region. The mass of a tachyon in the σ channel corresponds to the minus sign in front of the square root in (3). Its value is 1.3 GeV at $m_\sigma = 800$ MeV.

The term $\mathscr{L}_{\sigma\pi\pi}^{(4)}$ contains vertices $\sigma\pi\pi$ which, along with the analogous vertices in $\mathscr{L}_{\text{eff}}(\varphi)$, determine the decay width Γ_σ. An estimate of the vertices with the higher-order derivatives in (1) shows that perturbation theory becomes unreliable for Lagrangian (1) when the energies and momenta exceed 800 MeV. We can estimate the width Γ_σ for the dilaton mass $m_\sigma = 800$ MeV: $\Gamma_\sigma \simeq 440$ MeV. In this case, both the mass and the width Γ_σ agree with experiment.[4] The other decay channels described by

Lagrangian (1), $\sigma \rightarrow$gluons, $\gamma\gamma$..., will be discussed in a more detailed paper.

We can give the values of the condensates and of F_π which correspond to the mass $m_\sigma = 800$ MeV. With $F_\pi = 93$ MeV and $m_\pi = 140$ MeV, we used the following values for the condensates: $C_g = (335\ \text{MeV})^4$ and $\langle\bar{\psi}\psi\rangle = -(185\ \text{MeV})^3$. We have thus established that the gluon condensate plays the key role in the formation of the mass of scalar meson as a dilaton. We wish to stress that despite the existing opinion,[2] the scalar meson of the sigma model and the dilaton are not different particles. Within the framework of a common mechanism for the breaking of the chiral and conformal symmetries in QCD, the dilaton is a natural partner of the π meson. More-accurate predictions regarding the dilaton may become possible after a bosonization of the vector degrees of freedom of the quarks.

[1]M. Gell-Mann and M. Levy, Nuovo Cim. **16**, 705 (1958).

[2]H. Gomm, P. Jain, R. Johnson, and J. Schechter, Phys. Rev. D **33**, 801 (1986).

[3]A. A. Andrianov, V. A. Andrianov, V. Yu. Novozhilov, and Yu. V. Novozhilov, Preprint ITF-19R, Kiev, 1976; VANT ser.: Obshchaya i yadernaya fizika, No. 2(35), 41 (1986); Preprint , Carleton University 85/11, Ottawa, 1985.

[4]"Review of particle properties," Rev. Mod. Phys. **56** (1984).

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