Multiplicity dependence of the transverse momentum in a hydrodynamic theory of multiple production

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The transverse momentum $\bar{p}_\perp$ is calculated as a function of the secondary-particle multiplicity. The calculation is based on a hydrodynamic picture of collisions of gluon clusters of nucleons. There is a quantitative agreement with collider experimental data.

Collider experimental data[1] on the multiplicity dependence of $\bar{p}_\perp$ have recently attracted considerable interest. It has been found that $\bar{p}_\perp$ initially increases rapidly with increasing charged-particle density $N_{\rm ch}/\Delta y$ in the central rapidity region, $|\Delta y_c|\leqslant 2.5$, and then the curve becomes nearly flat. One possible explanation is based on a quark-hadron phase transition[2] in the flat part of the $\bar{p}_\perp(N_{ch})$ curve. That explanation, however, runs into the difficulty that the corresponding results for ISR energies differ both in the functional dependence and in the magnitude of $\bar{p}_\perp$. Other possible interpretations of the dependence[3–5] $\bar{p}_\perp(N_{\rm ch})$ run into the same difficulty. Fowler[5] has suggested that the inelasticity coefficient K falls off with increasing energy, from 0.5 at ISR energies to 0.3 at $\sqrt{s}=540$ GeV, since otherwise the increase in $\bar{p}_\perp(s)$ with the energy would be too rapid. The calculation of the functional dependence $\bar{p}_\perp(s)$ appears to be the primary difficulty in several models.

We previously[6] found $\bar{p}_\perp(s)$, by making use of the average inelasticity coefficient $K\simeq 0.5$. That approach, however, is not sufficient for calculating correlations between $\bar{p}_\perp$ and $N_{\rm ch}$: It is necessary to find the distribution of the gluon clusters of the colliding nucleons in the invariant mass M. This step requires finding the cluster distribution D_1 in the fraction of the energy of the clusters in the nucleon. We have found that distribution of three valence quarks, *uud,* in their total fraction of the energy in the nucleon, by using the Regge expressions[7,8] for the distributions of u and d quarks:

$$u(x) = c_1 x^{-\alpha_R(0)}(1-x)^{\alpha_R(0)-2\alpha_N(0)} \qquad (1)$$

where $\alpha_R(0)\simeq 0.5$ and $\alpha_N(0)\simeq -0.4$ are the intersections of the boson and nucleon Regge trajectories, and by using the normalization condition

$$c_1 = [B(\alpha_R(0),\ \nu_u+1)]^{-1}, \qquad \nu_u = \alpha_R(0) - 2\alpha_N(0).$$

The function $d(x)$ contains an extra power of $(1-x)$.

We wish to find the three-quark distribution function $G_{uud}(x_1,x_2,x_3)$, from which we find single-quark distributions. For example, we have

$$u(x) = \iiint G_{uud}(x_1,x_2,x_3)\delta(x-x_1)\theta(1-x_1-x_2-x_3)dx_1dx_2dx_3; \qquad (2)$$

there is a similar expression for $d(x)$. It can be shown that equations of the type in (2) are satisfied by the function G_{uud} in the form of a product of unknown functions, $\varphi_u(x_1)\varphi_u(x_2)\varphi_d(x_3)$. For the functions φ_u and φ_d we find a system of two nonlinear integral equations, which has been solved through an expansion of the functions φ_u and φ_d in power series in $\xi = (1-x)$ (with a maximum error $\sim 10\%$ at small values of x). The solution (in numbers) is

$$G_{uud} = \frac{0,51(1-0,45x_1+0,1x_1^2)(1-x_1)^{1,3}}{x_1^{1/2}} \prod_{i=2}^{3} \frac{(1-x_i)^{0,3}}{x_i^{1/2}} (1-0,62x_i+0,14x_i^2). \tag{3}$$

We can then find $f(x)$, the distribution of the three valence quarks in the total fraction of their energy in the proton, x:

$$f(x) = \iiint G_{uud}\, x_1, x_2, x_3)\delta(x - x_1 - x_2 - x_3)dx_1dx_2dx_3. \tag{4}$$

In this letter we are assuming that the leading particles form from the valence quarks of the colliding hadrons. Clusters consisting primarily of gluons interact strongly and determine the multiplicity of hadrons in the central region. The distribution (D_1) of clusters in their total fraction of the energy is given by

$$D_1(x) = f(1-x). \tag{5}$$

A calculation yields the following expression for the average fraction of the energy of the clusters:

$$\bar{x} = \int_0^1 xD_1(x)\,dx \approx 0,51.$$

The distribution of the gluon clusters in the invariant mass M is given by

$$D(\mu) = \iint_0^1 D_1(x_1)D_1(x_2)\delta(x_1x_2 - \mu)dx_1dx_2, \tag{6}$$

where $\mu \equiv M^2/s$. Omitting the intermediate analytic and numerical calculations, we show the function $D(\mu)$ in Fig. 1.

The experimental data of Ref. 9 show that the distribution of leading protons in the fraction of their energy in the region $0.1 \leqslant x \leqslant 0.9$ differs only slightly from a flat distribution, $f(x) \approx$ const (at ISR energies). Interestingly, according to (6), this circumstance leads to a function $D(\mu) \simeq \ln 1/\mu$, which is similar to that which is found in our model (Fig. 1). However, in, say, the valon model,[10,11] i.e., the model of constituent quarks carrying all the energy of the nucleon, $D(\mu)$ is a function with a maximum [and $D(0)=0$]:

$$D(\mu) = \left(\frac{105}{16}\right)^2 \mu^{1/2}\left[(1+4\mu+\mu^2)\ln\frac{1}{\mu} - 3(1-\mu^2)\right].$$

A corresponding calculation here leads to underestimates of the inelasticity coefficient and also of N_{ch} and $\bar{p}_\perp$ (for the ISR and the collider).

In collisions of gluon clusters of nucleons, a quark-gluon plasma forms in the

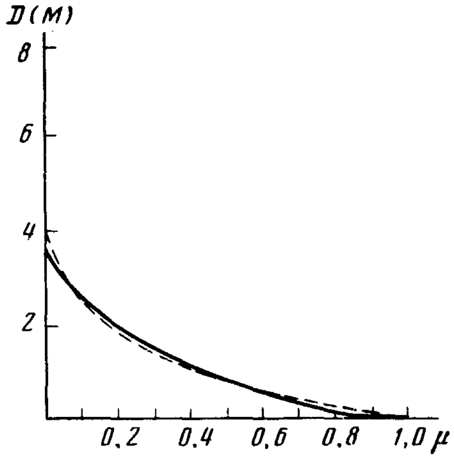

FIG. 1. Solid line—the function $D(\mu)$ in the model of the present paper; dashed line—the function $\ln(1/\mu)$.

compressed volume; a fraction $M/\sqrt{s}$ of the energy is expended on the formation of this plasma. The expansion of the plasma and the formation of hadrons are described by a hydrodynamic picture.[6] Knowing the distribution $D(\mu)$, we can calculate N_{ch}, $\bar{p}_\perp$, the dispersion, and other characteristics. These calculations will be presented in a separate paper. It should be noted that the multiplicity distribution at $\sqrt{s} = 540$ GeV at high multiplicities $N_{ch} > 2\overline{N}_{ch}$ is described well not by a Poisson distribution for each value of μ (as at ISR energies) but by a Bose-Einstein distribution $p_n^{(K)}$ of quanta with K thermal sources at $K \simeq 6$. With increasing energy, the number K decreases. The difference from a Poisson distribution, however, has essentially no effect on the functional dependence $\bar{p}_\perp(N_{ch})$ (Fig. 2).

The center of mass of the gluon clusters of nucleons of different fractions of the

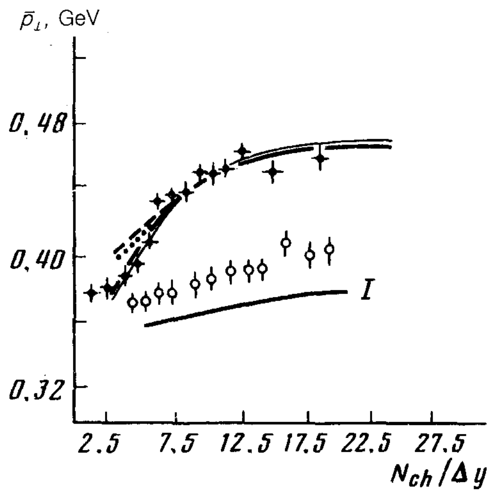

FIG. 2. Solid line—the function $\bar{p}_\perp(N_{ch})$ for $p_n^{(6)}$ with allowance for the rapidity shifts; thin solid line—the same function, for a Poisson distribution (collider); dashed and dotted lines—the same functions, respectively, without allowance for rapidity shifts; I—the function $\bar{p}_\perp(N_{ch})$ for $\sqrt{s} = 63$ GeV; ●—experimental data for $\sqrt{s} = 540$ GeV; O—experimental data for $\sqrt{s} = 63$ GeV.

energy, x_1 and x_2, is in motion in the c.m. frame of the initial nucleons; this circumstance complicates a calculation of $\bar{p}_\perp(N_{ch})$. Secondary hadrons from the edges of the rapidity distribution for the clusters, where the values of $\bar{p}_\perp(y)$ are smaller than at the center (because of the decaying temperature profile,[6,12] enter the central rapidity region, $|y_c| \leqslant 2.5$. The result is a good agreement between the value of $\bar{p}_\perp(N_{ch})$ and the collider experimental value at the left edge of the curve (at small values of $N_{ch}/\Delta y$). The expression for $\bar{p}_\perp(N_{ch})$ takes the following form when we take this motion into account, i.e., when we take the rapidity shifts into account:

$$\bar{p}_\perp(N_{ch}) = \frac{\int_0^1 d\mu \int_\mu^1 \frac{dx_1}{x_1} D_1(x_1) D_1(\mu/x_1) p_n^{(6)} [\bar{N}(\mu, x_1)] \bar{p}_\perp(\mu, x_1)}{\int_0^1 d\mu \int_\mu^1 \frac{dx_1}{x_1} D_1(x_1) D_1(\mu/x_1) p_n^{(6)} [\bar{N}(\mu, x_1)]} \qquad (7)$$

Here $\bar{N}(\mu, x_1)$ and $\bar{p}_\perp(\mu, x_1)$ are the values of $\bar{N}_{ch}$ and $\bar{p}_\perp$ in the region $|y_c| \leqslant 2.5$, calculated with allowance for the rapidity shifts. The results are shown in Fig. 2. We see that these shifts reduce the value of $\bar{p}_\perp(N_{ch})$ for the collider from 0.4 to 0.37 GeV/c at the left edge (at $N_{ch}/\Delta y \simeq 2.5$). At large multiplicities N_{ch}, the effect of the shifts is inconsequential.

At ISR energies the transverse momentum $\bar{p}_\perp(y)$ varies slowly,[6,12] so that the curve $\bar{p}_\perp(N_{ch})$ is flatter. For the energy $\sqrt{s} = 63$ GeV at $|y_c| \leqslant 2$, the curve falls slightly below the experimental data. A possible explanation is that the experimental values of $\bar{p}_\perp$ are $\sim 5\%$ too high, as mentioned in Ref. 13 [*sic*].

In summary, the hydrodynamic theory gives a quantitative description of the correlations between $\bar{p}_\perp$ and the multiplicity at various energies.

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