Optimum current density profile in a tokamak

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A series of experiments on electron-cyclotron-resonance heating of the plasma in the T-10 tokamak shows that as the ratio of the plasma density to the current is increased, the radial profile of the current approaches a limiting profile. This limiting profile can be found from a simple variational principle corresponding to a minimum of the energy of the plasma and of the poloidal magnetic field at the given current.

The current in a tokamak plasma is maintained by the vortical electric field E, while the radial profile of the current density J depends on the profile of the electron temperature T_e by virtue of the relation $J = \sigma E$ and the expression for the electrical conductivity, $\sigma = \sigma_0 T_e^{3/2}$. It might thus appear that the profile $J(r)$ would be determined by the profile $T_e^{3/2}(r)$, would depend on the balance between the heating and the heat loss due to thermal conductivity, and could differ from one device to another. Actually, however, the profiles $T_e(r)$ in different tokamaks are very similar to each other. Coppi[1] was the first to call attention to this "profile consistency." The consistency of the profiles $J(r)$ and $T_e(r)$ is seen most vividly in the largest devices, the JET[2] and the TFTR.[3] In the TFTR, for example, the $T_e(r)$ profile retains its universal shape during auxiliary heating at the periphery, with a power input up to $r/a \cong 0.8$, where a is the minor radius of the plasma. A similar result had been observed earlier in the T-10 during electron-cyclotron-resonance (ECR) heating of the plasma.[4] It has recently been studied in more detail in a series of experiments in the T-10 tokamak at various values of the current and the plasma density, in an experiment in which the power from the gyrotrons could be pumped into the plasma either at its center or at its periphery. The experiments have shown that during ohmic heating of the plasma the radial profiles of the temperature, $T_e(r)$, and the current density, $J(r)$, approach a "limiting profile" as the ratio $\bar{n}/J_p$ is raised, where $\bar{n}$ is the average plasma density, and J_p is the current. There is also a tendency for this profile to persist during auxiliary heating: A change in the power input profile does not change the limiting profile.

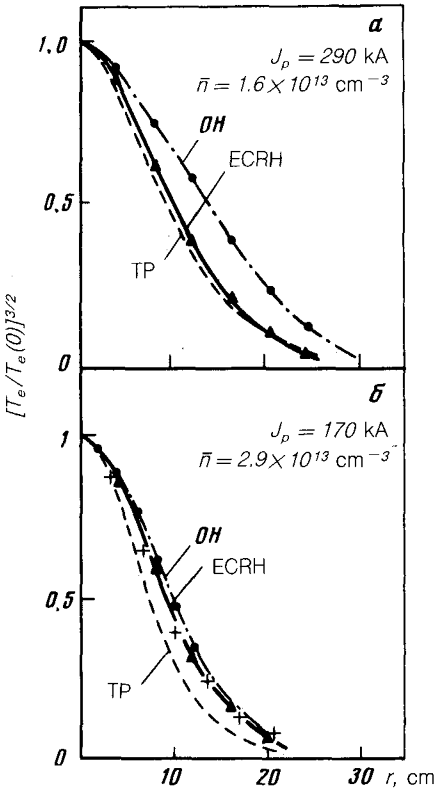

FIG. 1. Profiles of the electron temperature, $[T_e(r)T_e(0)]^{3/2}$, during ohmic heating (OH) and during electron-cyclotron-resonance heating (ECRH). a—$J_p = 290$ kA, $\bar{n} = 1.6 \times 10^{13}$ cm^{-3}; b—$J_p = 170$ kA, $\bar{n} = 2.9 \times 10^{13}$ cm^{-3}; TP—theoretical profile calculated under the assumption $k_e^{\rm OH} = k_e^{\rm ECRH}$, where k_e is the electron thermal conductivity; + —optimum profile (3), $J/J_0 = (1 + r^2/a_*^2)^{-2}$.

Instead, the profile of the thermal conductivity changes in such a way that the original profile is preserved. The situation is illustrated by Fig. 1, which shows profiles $T_e^{3/2}(r)$ during ohmic and ECR auxiliary heating for two values of the current, $J_p = 290$ kA and 170 kA. At $J_p = 290$ kA, auxiliary heating at the center sharpens the profile $T_e(r)$, and the profile continues to correspond to the calculated profile (calculated under the assumption that the electrical conductivity remains the same). At $J_p = 170$ kA, however, the limiting profile is reached even in the ohmic stage, and it is not changed by the auxiliary heating. As $\bar{n}/J_p$ is increased, the limiting profile is even more firmly entrenched.

While the limiting (optimum) profile is preserved over the entire plasma column, the confinement of electrons is no worse than during ohmic heating; when the limiting profile is disrupted, the confinement is degraded.

Theoretically, we would naturally link this formation of an optimum current profile to an instability of tearing modes,[3] which "sense" the entire radial profile of the current, and it would seem appropriate to describe this phenomenon in the approach of Taylor's theory[5] of relaxed states in an inverse pinch. Taylor's theory cannot be applied directly to a tokamak, since it assumes a complete reconnection of the lines of force. Furthermore, the energy of the toroidal magnetic field B is fixed in a tokamak. Let us assume that the relaxed state in a tokamak corresponds to a minimum of the energy (the energy of the poloidal magnetic field plus the thermal energy) at the given total current. We can then use the variational principle

$$\delta F \equiv \delta\left[\int \frac{B_\theta^2}{2}\, r dr + \frac{1}{\gamma - 1}\int P r dr + \lambda \int J r dr\right] = 0. \tag{1}$$

Here B_θ is the poloidal magnetic field, γ is the adiabatic index, P is the plasma pressure, J is the current density, and λ is a Lagrange multiplier. The quantity F may be thought of as a free energy whose minimum determines the most stable state. The variation of F should be carried out in perturbations qualitatively reminiscent of nonlinear tearing modes, which simultaneously flatten the $P(r)$, $J(r)$, and $q(r)$ profiles (q is the safety factor) near points of rational values of q, while the magnetic configuration remains the same far from magnetic islands. We accordingly set

$$B_\theta = d\psi/dr\ , \quad P = P(\mu), \quad J = J(\mu), \quad \mu \equiv q^{-1} = \frac{R}{B}\frac{1}{r}\frac{d\psi}{dr}\, .$$

in (1). A variation of (1) yields

$$\frac{d}{dr}\left[r^2\frac{B^2}{R^2}\mu + \frac{1}{\gamma - 1}\frac{dP}{d\mu} + \lambda\frac{dJ}{d\mu}\right] = 0. \tag{2}$$

Equation (2), along with the relation $J/J_0 = (1/2r)d/dr(r^2\mu)$, leads to the extremely simple solutions

$$P = P_0\mu^2\, , \quad J = J_0\mu^2\, , \quad \mu = \left(1 + \frac{r^2}{a_*^2}\right)^{-1}\, , \tag{3}$$

where we have chosen the integration constant to arrange $\mu(0) = 1$. Here

$$a_*^2 = R^2 J_p/I_0 = RJ_p/5B\ , \ . \tag{4}$$

where I_0 is the total number of ampere-turns of the toroidal magnetic field.

In a tokamak we have $J \sim T^{3/2}$, so that the optimum profile for J from (3) can be drawn as shown in Fig. 1. As we see, this profile agrees very well with the "limiting" profile for the T-10.

At the minimum, the free energy F is given by the following expression, after we substitute the value of λ:

$$F = \frac{B^2}{2R^2}\int [r^2 + a_*^2]\,\mu^2 r dr. \tag{5}$$

This free energy is independent of the thermal energy of the plasma, i.e., β_p. With a deviation from the optimum, however, F increases, and it does so more steeply, the larger the value of β_p. Upon a deviation from the optimum, we would naturally expect a degradation of the thermal insulation of the plasma, and this expectation correlates well with the results of the T-10 experiments. Specifically, as the ratio $\bar{n}/J_p$ (proportional to $\beta_p \sim nTa_*^2/J_p^2$ is increased, there is an increased tendency for the optimum profile to be preserved, but at the same time the conditions become more favorable for a degradation of the confinement upon a deviation from the optimum profile (in particular, when the limiter approaches the plasma). In other tokamaks also, the deviation of the scaling from that which prevails in the ohmic case increases with increasing β_p.

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