Weak localization in the magnetoresistance of a strongly anisotropic 2D system

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The localization-induced transverse and longitudinal magnetoresistances in a system of thick, weakly interacting metallic filaments arranged in the plane have been calculated. The open Fermi surface accounts for the fact that the diffusion-mode spectrum in the electron-electron channel is described by Mathieu's equation. The model proposed here can be used to describe kinetic phenomena in the dislocation layer at the cleavage boundary of the bicrystals with small intergrowth angles.

The study of two-dimensional localization has recently produced considerable interest in the high-conductivity layers situated in the intergrowth plane of germanium bicrystals.[1,2] A suitable model which describes the localization at $T = 0$ was studied by us in part previously.[3,4] It was suggested in Ref. 5 that such a layer be viewed as a quasi-one-dimensional system in the plane in which the edge dislocations are the one-dimensional metallic filaments.[5] The size of these dislocations is on the order $\rho = 30$–40 Å and the average spacing between them is

$$d = b/2\sin\frac{\theta}{2},$$

where $b \simeq 5$ Å, and θ is the intergrowth angle. The electron filling of the layer is determined by both the broken-bond potential and the screening potential of the holes. At small intergrowth angles we can have a situation in which $w_\perp \ll \epsilon_F \lesssim w_\parallel$, where $w_\parallel$ and $w_\perp$ are the resonance integrals directed along the dislocation tubes and perpendicular to them, where $w_\perp \simeq w_\parallel \exp(-d/\rho)$. The Fermi surface will then be, in contrast with the purely two-dimensional case in which $w_\perp \simeq w_\parallel$, a free surface $[\epsilon - \epsilon_F = v_F(|p_\parallel| - p_F) - w_\perp \cos p_\perp d]$, and the physical situation will thereafter depend on the relationship between the amplitude of the surface undulation and the impurity-induced diffuseness of the surface, i.e., the dimensionless parameter $w_\perp \tau$, where τ is the Boltzmann time of scattering by impurities, and $\epsilon_F \tau \gg 1$. At $w_\perp \tau \gg 1$ we are in fact dealing with a two-dimensional, although strongly anisotropic, localization. However, the attendant parametric dependence of the localization length, for example, differs from such a dependence for the anisotropic system with a closed Fermi surface.[3,4] In the other limit, $w_\perp \tau \ll 1$, the localization is one-dimensional is nature.

A disruption of a weak localization is caused by a magnetic field. The first measurements of the magnetoresistance in various systems and in various field configurations were carried out at the first-correction level in Refs. 6 and 7. Although this problem is now being studied extensively, in all cases the Fermi surface is assumed closed, so that a function of the type $\ln(H/H_0)$ always holds in the two-dimensional case. We will show below that a different result is obtained in the case of an open Fermi surface. The first localization correction to the conductivity, which comes from the diffusion mode in the electron-electron channel, can be written in the form

$$\sigma_j = \sigma_j^0 \left(1 - \frac{1}{N} Z\right); \qquad \sigma_\parallel^0 = e^2 \frac{2lN}{\pi d}; \qquad \sigma_\perp^0 = e^2 (w\tau)^2 N \frac{d}{l}, \tag{1}$$

$$Z = \frac{v_F d}{V} \sum_n \frac{1}{\Omega + \epsilon_n}, \tag{2}$$

where $N = Sp_F^2$, S is the cross section of the filament [we assume here that $N \ll \epsilon_F/w$ (here and below the symbol $\perp$ next to w is dropped)], and V is the "volume" of the system, where $V = L_\perp L_\parallel$. In (2)

$$\Omega(T) = \frac{1}{\tau_{in}} < \frac{1}{\tau}$$

is the inelastic-collision frequency, and ϵ_n is the spectrum of the indicated mode. In the presence of a magnetic field, this spectrum can be determined from the equation

$$\left[-D_\parallel \frac{\partial^2}{\partial x^2} + D_\perp \left(1 - \cos\left(d\left(\frac{1}{i}\frac{\partial}{\partial y} - \frac{2eH}{c} x\right)\right)\right)\right]\Psi_n = \epsilon_n \Psi_n, \tag{3}$$

where $D_\parallel = v_F l$ and $D_\perp = w^2 \tau$ are the longitudinal diffusion coefficient (the x axis) and the transverse diffusion coefficient, and $l = v_F \tau$. Equation (3) can be derived by making use of the specific results for the diffusions at[8] $H = 0$ and by treating the field H in the semiclassical approximation in the standard way adopted in the superconductivity theory.[9] If we assume that $\Psi(x,y) = e^{iq_\perp y}u(x)$, Eq. (3) becomes the familiar Mathieu equation

$$\left[- D_\parallel \frac{\partial^2}{\partial x^2} + D_\perp\left(1 - \cos\left(q_\perp d - \frac{2dx}{l_H^2}\right)\right)\right]u = \epsilon u , \tag{4}$$

in which the magnetic length $l_H^2 = e/Hc$ is introduced and the allowable fields are restricted by the condition $l_H \gg \sqrt{s}$. The open Fermi surface distinguishes Eq. (4) from the standard oscillator equation. Equations (2) and (4), after rendering them dimensionless, can be written in the form

$$Z = 2N_m \int \frac{dE}{\pi} \rho(E) \frac{1}{E + N_m^2 \Omega\tau} , \tag{5}$$

where $N_m = l_H^2/dl$, and $\rho(E)$ is the spectral state density of the equation

$$\left[4 \frac{\partial^2}{\partial t^2} + E + J(\cos t - 1)\right] u = 0, \tag{6}$$

where $J = (w\tau N_m)^2$. If we assume that $w\tau \ll 1$, the important parameter of the problem will be J. In the limit $J \to 0$ (this case corresponds to strong fields $N_m < 1/w\tau$ and allows a transition to independent filaments, $w = 0$) spectrum (6) is $E = 4q^2$ and $\rho(E) = 1/4\sqrt{E}$. Incorporation of a weak perturbation in J accounts for the displacement of the front end of the spectrum $E = 4q^2 + J - \frac{1}{2}J^2 + o(J^3)$ and for the appearance of band gaps at $E_n = n^2$ of width on the order[10] $\Delta E_n \simeq J^n$. Ignoring the last effect, we find

$$Z(H, T) = \frac{1}{2} \frac{1}{\sqrt{\Omega\tau + (w\tau)^2}} \left(1 + \frac{(w\tau)^2}{\Omega\tau + (w\tau)^2} \left(\frac{l_H^2 w\tau}{2dl}\right)^2 + o(J^2)\right), \tag{7}$$

i.e., the magnetoresistance in the fields

$$l_H^2 < d\frac{v_F}{w}$$

becomes saturated.

At $J \gg 1$ and $E < J$, Eq. (6) becomes the oscillator equation

$$\left[4 \frac{\partial^2}{\partial t^2} + E - \frac{J}{2}t^2\right] u = 0. \tag{8}$$

In this case, the spectrum is comprised of strongly degenerate, discrete levels and $\rho(E) = 1/2\delta(E - E_n)$, where $E_n = (n + 1/2)\sqrt{8J}$. Since allowance for tunneling between the degenerate states in (8) leads to a slight Harper broadening of the levels $[\sim \exp - (J - E)]$ in comparison with the level spacing, this broadening can be discarded so long as $E_n < J$ or $n < N_c$, where

$$N_c \simeq \sqrt{\frac{J}{8}} = \frac{l_H^2 w}{\sqrt{8}\, d v_F}$$

At $E > J$ the reverse situation occurs. In this case we have broad allowed energy bands and narrow forbidden bands and we can again roughly assume that the spectrum is continuous with $\rho(E) = 1/4\sqrt{E}$. As a result, we find

$$Z(H, T) = \frac{1}{\sqrt{8}\pi w \tau} \ln \frac{\Omega\tau + (w\tau)^2}{\Omega\tau + 2(w\tau)^2/\sqrt{2J}} + \frac{1}{2} \frac{1}{\sqrt{\Omega\tau + (w\tau)^2}} \, . \tag{9}$$

It follows that the magnetic field $l_H^2\, dv_F/w$ perpendicular to the layer is appreciable if $\Omega(T) \ll w^2\tau$. We then can write

$$Z(0, T) - Z(H, T) = \frac{1}{2\sqrt{2}\pi w\tau} \ln\left(1 + \sqrt{2}\frac{w}{\Omega}\frac{ld}{l_H^2}\right). \tag{10}$$

At temperatures where $w^2\tau \ll \Omega(T)$, Z varies as a function of H within

$$\Delta Z \simeq \frac{3}{16} \frac{(w\tau)^2}{(\Omega\tau)^{5/2}} \, . \tag{11}$$

In the case $1 \ll w\tau \ll \epsilon_F\tau$, the first correction can be calculated directly and the result can be written in the form

$$Z(H, T) = \frac{1}{4\pi}\frac{d}{l_\perp} \ln\left(1 + 1/\left(\Omega\tau + \frac{l^2}{l_H^2}\right)\right), \tag{12}$$

where $l^2 = l_\parallel l_\perp$ and $l_\perp^2 = (wd\tau)^2/2 \ll l_\parallel^2 = (v_F\tau)^2$, which is similar to the standard two-dimensional result. Accordingly, at $\tau w \simeq 1$ there is a crossing from a one-dimensional behavior to a three-dimensional behavior.

Finally, let us consider the magnetic field within the individual filaments.[7] Such an analysis can be carried out through the following substitution in the present equations:

$$\Omega\tau \to \Omega\tau + b\frac{l^2 S}{l_H^4},$$

where b is a number on the order of unity. The magnetoresistance of the field directed parallel to the layer is attributable entirely to this magnetic field, so that

$$Z(H, T) = \frac{1}{\sqrt{2}\pi w\tau} \ln\left(1 + \frac{w\tau}{\left(2\Omega\tau + b\frac{l^2 S}{l_H^4}\right)^{1/2}}\right), \tag{13}$$

$$Z(H, T) = \frac{1}{4\pi}\frac{d}{l_\perp} \ln\left(\max\left(\Omega\tau, l_\parallel^2 S/l_H^4\right)\right), \tag{14}$$

where (13) is for $w\tau \lesssim 1$ and (14) is for $w\tau \gg 1$. Since we restricted the discussion above to the consideration of only the first localization correction, the applicability of the results is restricted by the condition that Z/N be small. It should be borne in mind that the standard small parameter $1/\epsilon_F\tau \ll 1$ in this case is replaced by $1/Nw\tau$ or $d/Nl_\perp$, so that this correction is much larger than the standard correction. The experimental situation in bicrystals[1/2] implies that the e-e interaction must be taken into account. We took it into account in the corrections to the stable density.[11] The role the e-e interaction plays in the magnetoresistance of such systems requires further study.

We thus see that the open nature of the Fermi surface significantly affects all the results, especially if the amplitude of the undulations is small in comparison with the impurity scattering, $w\tau < 1$. Such a situation can be arranged experimentally by choosing bicrystals with a moderate intergrowth angle, $\theta < 7$–$10°$, with $d > 40$ Å.

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