Orbital angular momentum and friction force between vortices and the normal component in ^{3}He-A

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The friction force is derived from hydrodynamics. The derivation indicates that the orbital angular momentum, which determines the inertia of the orbital motion, strongly affects the critical behavior of the friction force in the A phase. The values of this angular momentum which follow from the existing elementary theories are discussed.

Experiments carried out to measure the Hall-Vinen dissipative parameter (B) for the longitudinal friction force caused by vortices have shown that the temperature dependence of B near T_c in the A phase of ^{3}He is not the behavior $B \propto (1 - T/T_c)^{-1/2}$ which follows from Kopnin's theory.[1] Hall[2] explained this discrepancy in terms of an effect of an internal orbital angular momentum, which would have to be on the order

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of $\rho_s \hbar/M$, where M is the mass of a Cooper pair, in order to achieve agreement with experiment. Hall's hydrodynamic theory, however, leans on several assumptions of dubious validity.[3] In the present letter we offer a derivation of the friction force from hydrodynamics which does not use Hall's assumptions.[2] Although Hall's idea of a significant effect of the orbital angular momentum on the friction force near T_c is generally confirmed, the angular momentum itself turns out to have an origin different from that which appeared in Hall's theory. Specifically, it has now been established[4,5] that an internal orbital angular momentum is not a well-defined quantity; it takes on different values, depending on how it is determined theoretically and experimentally. According to our theory, the friction force depends on the dynamic orbital angular momentum L, which is a coefficient in the equation of motion for the orbital vector $\mathbf{l}$ [see Eq. (1) below] and which determines the orbital inertia. This point was ignored by Hall[2] as well as by Kopnin[1] because of the belief, dating back to Refs. 6 and 7, that the elementary theory for the A phase predicts vanishingly small values of the dynamic orbital angular momentum L—values proportional to the small parameter $(T_c/\epsilon_F)^2$. In contrast, the present study shows that only the dynamic angular momentum L, and nothing else, could be large enough (on the order of $\rho_s \hbar/M$) to explain the difference discovered by Hall between the experimental value of B and that predicted by Kopnin's theory.

The derivation of the friction-force parameters B and B' for a nonsingular vortex in the A phase which we offer below differs from Kopnin's derivation[1] in the following ways: 1) The derivation is made from a hydrodynamic theory, not from the kinetic equation which Kopnin constructed exclusively for the vicinity of T_c. Consequently, the range of applicability of the derivation is not restricted to the Ginzburg–Landau region. 2) We retain the orbital-inertia term in the equation of the orbital motion for L. We start from the following equations of motion for an incompressible fluid:

$$L\left[\mathbf{l},\frac{\partial \mathbf{l}}{\partial t}+(\mathbf{v}_n\vec{\nabla})\mathbf{l}\right]-\mu\left(\frac{\partial \mathbf{l}}{\partial t}+(\mathbf{v}_n\vec{\nabla})\mathbf{l}\right)=\frac{\partial H}{\partial \mathbf{l}}-\nabla_j\frac{\partial H}{\partial \nabla_j \mathbf{l}}+\frac{\partial H}{\partial \nabla_j \Phi}[\nabla_j \mathbf{l},\mathbf{l}]\ , \tag{1}$$

$$0=-\nabla_j\frac{\partial H}{\partial \nabla_j \Phi}\ . \tag{2}$$

Here H is the energy, Φ is the phase of the condensate, and μ is the Cross viscosity. We assume that a vortex is moving at a velocity v_L, which is assumed to be small. The terms on the left side of (1) can then be dealt with by perturbation theory, and we can write $(\partial \mathbf{l}/\partial t)+(v_n\cdot\nabla)\mathbf{l}\approx((\mathbf{v}_n-\mathbf{v}_L)\cdot\nabla)\mathbf{l}_0$, where $\mathbf{l}_0$ corresponds to the equilibrium texture for a vortex which is at rest with respect to the normal fluid. The right sides of (1) and (2) can be expanded in $\mathbf{l}'=\mathbf{l}-\mathbf{l}_0$ and $\Phi'=\Phi-\Phi_0$. As a result, Eqs. (1) and (2) become linear, inhomogeneous differential equations for $\mathbf{l}'$ and Φ'. To find the conditions under which they can be solved, we need to multiply Eqs. (1) and (2) by $(\mathbf{t}\nabla)\mathbf{l}_0$ and $(\mathbf{t}\cdot\nabla)\Phi_0$, respectively ($\mathbf{t}$ is an arbitrary translation vector), add them, and integrate them over the entire space. In this process, the integral of the right sides of (1) and (2) can be reduced, by an integration by parts, to integrals over a remote surface around the vortex. As a result, we find

$$\int d\mathbf{r}(\mathbf{t}\cdot\vec{\nabla})\mathbf{l}\{L[\mathbf{l},((\mathbf{v}_n-\mathbf{v}_L)\cdot\vec{\nabla})\mathbf{l}]-\mu((\mathbf{v}_n-\mathbf{v}_L)\vec{\nabla}\cdot)\mathbf{l}\}$$
$$=-\frac{\hbar}{M}\int dSn_i\{\lambda_i'(\mathbf{t}\cdot\vec{\nabla})\Phi-\Phi'(\mathbf{t}\cdot\vec{\nabla})\lambda_i\}. \qquad (3)$$

Here n_i are the components of the unit vector normal to the remote surface, $\boldsymbol{\lambda}=\partial H/\partial\nabla\Phi$ is the superfluid current in the coordinate system moving at the normal velocity $\mathbf{v}_n$, and we are omitting the index 0 from the quantities which correspond to the ground state of a vortex at rest. Following Kopnin,[1] we assume that the vortex is axisymmetric and that the orbital vector $\mathbf{l}$ is parallel to the axis of the vortex far from the vortex. Condition (3) must be satisfied for any translation vector. Carrying out the integration over the surface in (3), we find the following condition for the absence of secular terms—a condition which relates the velocities $\mathbf{v}_L$, $\mathbf{v}_n$, and $\mathbf{v}_s=\hbar/M\nabla\Phi$:

$$\mathbf{v}_L=\mathbf{v}_s+\frac{\rho_n}{2\rho}B[\mathbf{z},\mathbf{v}_n-\mathbf{v}_s]+\frac{\rho_n}{2\rho}B'(\mathbf{v}_n-\mathbf{v}_s), \qquad (4)$$

where

$$B=\frac{B_k}{1+G^2}, \qquad B'\doteq\frac{2\rho}{\rho_n}-B_K\frac{G}{1+G^2},$$

$$B_K=\frac{2\rho\rho_s}{\rho_n}\frac{\kappa}{\gamma\mu}\sim\frac{7}{\gamma}\left(1-\frac{T}{T_c}\right)^{-1/2}, \qquad G=\frac{\rho_n}{2\rho\rho_s}\frac{ML}{\hbar}B_K, \qquad (5)$$

and the direction of the circulation κ is determined by the unit vector $\mathbf{z}$. The number γ depends on the structure of the vortex; for an axisymmetric vortex it is determined by the integral

$$\gamma=\pi\int_0^\infty rdr\left(\frac{\partial\mathbf{l}}{\partial r}\frac{\partial\mathbf{l}}{\partial r}+\frac{l_\perp^2}{r^2}\right), \qquad (6)$$

where $l_\perp$ is the component of $\mathbf{l}$ in the plane normal to the vortex axis. According to Hall,[2] we have $\gamma=\pi^3/2$ or $\pi^2/3$ for different models of an Anderson-Toulouse two-quantum nonsingular vortex. The quantity B_K is the value of B found by Kopnin.[1] The orbital-inertia effect is determined by the parameter G, which is proportional to $(1-T/T_c)^{-1/2}$ if L is proportional to ρ_s. This effect determines the temperature dependence $B\sim(1-T/T_c)^{1/2}$ in the limit $T\to T_c$. If the condition $L\ll\rho_s\hbar/M$ holds, however, as is predicted by the elementary theory of Refs. 6 and 7, the orbital inertia is negligible at the values of T_c-T which can be attained experimentally.

The distinction between the results of the present study and those found by Hall[2] reduces to the particular angular momentum L which is to be substituted into the expressions for B and B'. Hall's result can be reproduced by using the substitution $L=(\lambda+\rho_s-\rho_L)\hbar/M$, where the effective density ρ_L is related to an orbital angular momentum introduced by Hall and Hook[5] in the hydrodynamic equations, while the effective density λ determines the orbital inertia in Hall's interpretation. For this reason, our results can be transcribed into Hall's notation by the substitution $L=\lambda\hbar/M$. As we have already noted, however, Hall ignored the orbital inertia, setting $\lambda=0$

in his calculations. The hydrodynamic derivation of expressions for B and B' given above is more rigorous than Hall's derivation, since it does not lean on Hall's assumptions regarding the nature of the mutual friction force. The difference in the results, on the other hand, tells us the extent to which these assumptions correspond to the nature of the mutual friction in the A phase.

For example, the discrepancy which Hall identified between experiment and the theory of the friction force ignoring the orbital angular momentum indicates that the dynamic orbital angular momentum L may not be as small as is predicted by the elementary theory of Refs. 6 and 7. This important conclusion will of course require a further experimental test, as close to T_c as possible, especially since previous experiments on orbital dynamics[8] have revealed no orbital-inertia effect. However, the elementary theory of Refs. 6 and 7 has its own vulnerabilities, in our opinion, linked with the determination of the contribution of "boojums" i.e., points on the Fermi surface where the gap in the quasiparticle spectrum vanishes. We will not go into a discussion of these difficulties here; they do indicate that the question of the value of L in the elementary theory is not closed. Some recent studies,[9,10] have been aimed at an exact solution of the quantum-mechanical problem in the boojum region, but the problem has not yet been solved completely.

We note in conclusion that L could also be determined experimentally through an observation of Kelvin waves in the rotating A phase,[11] since these waves are orbital waves of a periodic **l** texture of a rotating fluid.

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