Contribution to the theory of nonlinear effects in the electric conductivity of metallic junctions

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A rigorous solution is presented of the problem of the electric conductivity of normal contacts of the constriction type in the limit when $a \ll l$ (a is the radius of the contact and l is the mean free path). The distribution of the electric field in the constriction region and the nonlinear singularities on the current-voltage characteristics are determined in an energy region on the order of the Debye phonons and of the Fermi energy. An analogy is noted between the properties of pure microbridges and tunneling junctions.

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Experiments[1,2] have shown that an investigation of the electric conductivity of microscopic point contacts makes it possible to study the electron–phonon interaction in normal metals, namely, to reconstruct the form of the function $\alpha^2(\omega)F(\omega)$, which is the product of the density of the phonon states by the square of the matrix element of the electron–phonon interaction. A qualitative interpretation of the phenomena that occur in the contact is relatively simple. At the same time a quantitative elucidation of the experimental data calls for knowledge of the spatial distribution of the electric fields produced in the constriction and of the form of the nonequilibrium, and also strongly inhomogeneous, distribution function. This raises the question of the degree to which the observed characteristics depend on the actual geometry of the constriction, which in most cases is not known exactly. All this makes it necessary to develop a quantitative theory that takes into account the spatial inhomogeneity of the problem.

We choose as a model of the junction a hole of radius a in an impermeable partition Σ (the $z=0$ plane) separating two bulky metallic half-spaces (Fig. 1). It is required to solve, under the condition $l \gg a$, the kinetic equation for the distribution function $f(\mathbf{p},\mathbf{r})$

$$\mathbf{v}\frac{\partial f}{\partial \mathbf{r}} + e\mathbf{E}\frac{\partial f}{\partial \mathbf{p}} = 0 \cdot \tag{1}$$

Far from the constriction we have an equilibrium distribution function $f=f_0(\epsilon_p)$. The distribution of the field $\mathbf{E}=-\nabla\Phi$ is obtained from the electroneutrality condition

$$e\int d\mathbf{r}_{\mathbf{p}}\,[\,f(\mathbf{p},\mathbf{r}) - f_0(\epsilon_{\mathbf{p}})\,] = 0 \tag{2}$$

with account taken of the boundary condition $\Phi=\pm V/2$ at infinity ($z\to\pm\infty$). V is the potential difference between the ends of the junction. We obtain the solution of the first-order Eq. (1) by the method of characteristics $f=F(\epsilon_p - e\int_L \mathbf{E}d\mathbf{l})$, where the integral is taken along the electron trajectory L. A solution that satisfies the necessary conditions is

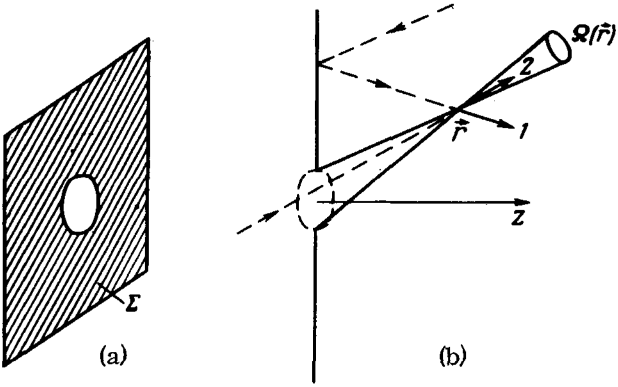

FIG. 1. Diagram of constriction (a) and electron trajectories near the hole (b). Trajectories of type 1 and 2 correspond respectively to electrons arriving at the point r from the region $z=+\infty(\eta=-1)$, and $z=-\infty(\eta=+1)$.

$$f = f_0\left(\epsilon_p + e\Phi(\mathbf{r}) + \frac{eV}{2}\eta(\mathbf{p},\mathbf{r})\right), \tag{3}$$

where the function $\eta(\mathbf{p},\mathbf{r})$ is defined as[1)]

$$\eta = \begin{cases} +1 & \mathbf{v} \in \Omega(\mathbf{r}) \\ -1 & \mathbf{v} \,\bar{\in}\, \Omega(\mathbf{r}) . \end{cases}$$

Here $\Omega(\mathbf{r})$ is the solid-angle element subtended by the hole when viewed from the point $\mathbf{r}$ (see Fig. 1). In view of the symmetry of the problem we obtain in the plane of the hole $\Phi=0$, and the function (3) becomes

$$f = f_0\left(\epsilon_p + \frac{eV}{2}\,\mathrm{sign}\, v_z\right). \tag{4}$$

The current density is given by the relation $\mathbf{j}=2e\int d\tau_{\mathbf{p}}\mathbf{v}f$. Substituting here expression (4), we obtain the total current

$$I = 2eS \int_{(v_z>0)} dS_F \frac{v_z}{v_\perp} \int d\epsilon_p \left[f_0\left(\epsilon_p - \frac{eV}{2}\right) - f_0\left(\epsilon_p + \frac{eV}{2}\right)\right], \tag{5}$$

where S is the area of the hole. The integral is taken over that part of the Fermi surface on which $v_z>0$.

At small V, expression (5) yields Ohm's law $I=V/R_0$, where the resistance is given by

$$\frac{1}{R_0} = \frac{e^2 S S_F}{(2\pi\hbar)^3} \langle \cos\theta \rangle \tag{6}$$

S_F is the area of the Fermi, the brackets $\langle\cdots\rangle$ denote averaging over S_F, and $\cos\theta = v_z/v_\perp$.

Expression (6) corresponds to Sharvin's formula,[3] according to which R_0 is equal to the resistance of a cylinder of area S and length l, namely $R_0 \sim l/\sigma S$, with R_0 independent of l. In the case of a quadratic dispersion law, Eq. (6) goes over into the expression obtained by Wexler.[4]

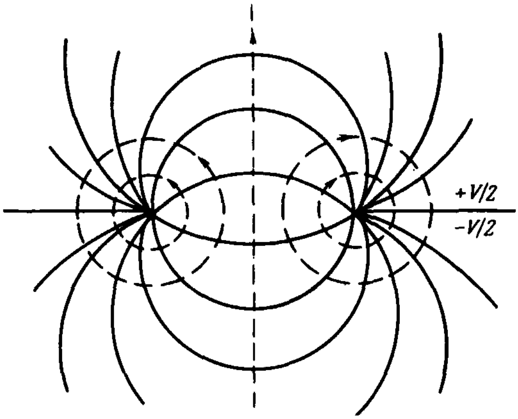

FIG. 2. Distribution of electric field near the hole.

The distribution of the field $\Phi(\mathbf{r})$ is obtained by substituting (3) in (2). At eV that are small in comparison with ϵ_F we obtain

$$\Phi(\mathbf{r}) = \frac{V}{2}\left[1 - \frac{1}{2\pi}\Omega(\mathbf{r})\right] . \qquad (7)$$

For example, on the symmetry axis we have $\Phi(z, p=0) = (V/2)(z/\sqrt{z^2+a^2})$. The equipotential surfaces, shown in cross section in Fig. 2, are the geometric loci of the points from which the opening is seen at a fixed solid angle Ω. The dashed lines in Fig. 2 represent the field intensities. The separation surface is equipotential and its potential on the corresponding sides is $\pm V/2$. As seen from the solution (7), the electric field is localized in a region of space on the order of the constriction radius a.

Expressions (5) and (6), as is clear from the derivation, do not depend on the shape of the constriction. The foregoing calculation pertained to the case of specular reflection of the electrons from the surface, but inasmuch as on Σ we have an equilibrium distribution function (3), the obtained solution is valid also for diffuse scattering.

Formula (5) pertains to the case $l \gg a$ at arbitrary V, and this expression describes the nonlinearities of the current-voltage characteristics of the bridge. Assuming for simplicity the function $\epsilon(\mathbf{p})$ to be isotropic, we obtain, taking into account the electron-phonon interaction in the Fröhlich model[5]

$$\frac{d^2 I}{dV^2} = \frac{e}{2R_0}\sum_a\left(\frac{p_{1a}}{v_{1a}} - \frac{p_{2a}}{v_{2a}}\right)\left[1 + \int_0^\infty d\omega\, \alpha^2(\omega)F(\omega)\left(\frac{1}{\omega + \frac{eV}{2}} + \frac{1}{\omega - \frac{eV}{2}}\right)\right]. \qquad (8)$$

Here $p_{\alpha 1,2}$ are the roots of the equation $\epsilon(p) = \epsilon_F \pm eV/2$. The quantity d^2I/dV^2 has singularities (jumps) at values $eV = 2\epsilon_F$, owing to the first factor in (8), as well as a characteristic variation near the maximum of the function $\alpha^2(\omega)F(\omega)$ on account of the quantity in the square brackets.[2)] The latter corresponds to the self-energy corrections to the electron dispersion law when account is taken of its interaction with the phonons. In the case of quadratic dispersion, we have $p_1/v_1 = p_2/v_2$ (at $eV < 2\epsilon_F$), so that these self-energy corrections (due to virtual phonons) are equal to zero, and in the case of a nonquadratic dispersion $\epsilon(p)$ they are proportional to V, i.e., they are odd in the voltage. These corrections can be observed in experiment, just as the CVC nonlinearities due to the terms a/l (real phonons) that are even in V.[1,2] In this case there is a deep analogy between pointlike and tunnel contacts. Taking into account the estimated path time $\tau \sim \omega_D^2/\max(T^3, \epsilon^3)$, we obtain at $\epsilon \sim \omega_D$ a condition on the contact dimension, $a \ll v_F/\omega_D \sim 10^{-5}$ cm. Neglect of inelastic electron-electron collisions in the study of the nonlinearities of the CVC in the region $eV \lesssim \epsilon_F$ is valid at $a \ll \lambda_F$, where $\lambda_F = h/p_F$ is the de Broglie wavelength of the electron.

For Yanson contacts obtained by soft breakdown of the dielectric[1,2] we have $a \sim 10$—100 Å, i.e., the last condition is satisfied for the case of metals of the bismuth type. A nonlinearity of this type can also be used to determine directly the Fermi energy of small groups of electrons in "standard" metals. Recently,[1,2] experiments were performed also on point contacts of a different type.[6]

In conclusion, we take the opportunity to thank I. K. Yanson for a discussion of this work and for useful remarks.

1)This expression is valid in weak fields $eV < \epsilon_F$. In a strong field, the trajectories become bent, a phenomenon similar to the change in the shape of the barrier by an applied voltage in a tunnel junction. This effect, however, does not exert a decisive influence on the formation of the nonlinear current-voltage characteristic (CVC) of the junction (see below).

2)The nonlinearity of the CVC of a point contact at $eV < \epsilon_F$ is due, according to[8], to the nonquadratic character of the dispersion law. If the Fermi surface is not spherical, additional singularities arise, due to the topological structure of the equal-energy surfaces and to its variation with increasing energy ϵ.

1I. K. Yanson, Zh. Eksp. Teor. Fiz. **66**, 1035 (1974) [Sov. Phys. JETP **39**, 506 (1974)].

2I. K. Yanson and Yu. N. Shalov, Zh. Eksp. Teor. Fiz. **71**, 286 (1976) [Sov. Phys. JETP **44**, 148 (1976)]; Fiz. Nizk. Temp. **3**, 99 (1977) [Sov. J. Low Temp. Phys. **1**, 48 (1977)].

3Yu. V. Sharvin, Zh. Eksp. Teor. Fiz. **48**, 984 (1965) [Sov. Phys. JETP **21**, 655 (1965)].

4G. Wexler, Proc. Phys. Soc. London **89**, 927 (1966).

5P. Allen, Phys. Rev. **B3**, 305 (1971); G. Grimvall, Phys. Kondens. Mater. **9**, 283 (1972).

6A. G. M. Jansen, F. M. Mueller, and P. Wyder, Direct measurement of electron-phonon coupling using point contacts, preprint, 1977.