

# Current generation in the interaction of a beam of accelerated multiply charged ions with a plasma

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It is shown that when a compensated beam of fast  $Z$ -ions is decelerated in a plasma, a strong electric current is produced and can be  $Z$  times larger than the current of the multiply charged ions.

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When a beam of fast multiply charged ions is decelerated in a plasma, an appreciable fraction of the momentum is transferred to the electrons, owing to the peculiarities of the Coulomb interaction. This leads to the appearance of an electron current that can, as will be shown below, exceed by one or two orders of magnitude the current of the multiply charged ions. The possibility of such a mechanism of amplifying the current in a plasma was pointed out to the author by H. Furth.

The present paper is devoted to a theoretical investigation of this phenomenon.<sup>1)</sup>

Consider a half-space  $x > 0$  filled with a plasma, into which a compensated beam of  $Z$ -ions of mass  $M_z$  enters with velocity  $v_0$  in the  $x$  direction. The concentration  $N$  of the electrons and ions in the plasma is much larger than the density of the  $Z$ -ions:  $N \gg N_z$ . Therefore the process of deceleration of the  $Z$ -ions in the plasma does not depend on the interaction between them, i.e., it is linear. Recognizing furthermore that we are considering here only fast multiply charged ions, whose velocity is of the order of the thermal velocity of the electrons, we can neglect in first-order approximation the motion of the basic ions of the plasma. The deceleration of the  $Z$ -ions, i.e., the decrease of their velocity  $v_z$  and of their concentration  $N_z$  with increasing depth of penetration of the beam into the plasma, is then described by the equation<sup>[1,3]</sup>

$$\frac{dv_z}{dx} = - \left[ \frac{M_z}{m} G(\gamma) + \frac{M_z}{M} \right] v_z, \quad N_z = N_{z0} v_0 / v_z; \\ \gamma = v_z (m/2T_e)^{1/2}, \quad G(\gamma) = \Phi(\gamma) - 2\pi^{-1/2} \gamma e^{-\gamma^2}, \quad \Phi(\gamma) = \frac{2}{\pi^{1/2}} \int_0^\gamma e^{-t^2} dt, \quad (1) \\ v_z = \frac{4\pi e^4 N Z^2 \ln \Lambda}{M_z^2 v_z^3}, \quad v_z \Big|_{x=0} = v_0.$$

This equation has a solution that defines in implicit form the function  $y(x)$ , i.e., the ion velocity  $v_z(x)$ , namely

$$\Psi(\gamma) = \Psi(\gamma_0) - \frac{\zeta}{L}, \quad \Psi(\gamma) = \int_0^{\gamma} \frac{\gamma^3 d\gamma}{G(\gamma) + m/M}, \quad \zeta = \int_0^x \frac{N(x)}{N_0} dx.$$

$$L = \frac{Ml}{mZ^2}, \quad l = T_e^2 / \pi e^4 N_0 \ln \Lambda, \quad \gamma_0 = v_0 (m/2T_e)^{1/2} \quad (2)$$

We now consider the excitation of the electron current. As seen from (2), the characteristic  $Z$ -ion deceleration length  $L$  in the plasma is much larger than the electron mean free path  $l$ . Therefore, to determine the perturbation of the electron distribution function we can confine ourselves to the locally homogeneous problem, assuming the velocity  $v_z$  and the concentration  $N_z$  to be specified at each point  $x$  in accordance with (1) and (2). The kinetic equation for the electrons is written in a coordinate system that moves with a velocity equal to the average velocity of the principle plasma ions. It is of the form

$$S_{ee} + S_{ei} = -S_{ez}, \quad (3)$$

where  $S_{ee}$ ,  $S_{ei}$ , and  $S_{ez}$  are the integrals of the electron collisions with one another, with the principal plasma ions, and with the  $Z$ -ions.

The integral  $S_{ez}$  is the source of the perturbation of the electron distribution function. We expand, as usual, the distribution function in a series in the Legendre polynomials  $P_k(\cos\theta)$ :

$$f(v) = f_0(v) + f_1(v)\cos\theta + f_2(v)P_2(\cos\theta) + \dots,$$

where  $\theta$  is the angle with the  $x$  axis. Starting with the Landau integral,<sup>[4]</sup> we arrive at the following expression for  $S_{ez}$ :

$$\begin{aligned} S_{ez0} &= -\frac{v_z^2}{3v^2} \frac{\partial}{\partial v} \left\{ v^2 \frac{\partial f_0}{\partial v} \left[ \begin{array}{l} \nu_{ez}(v), \quad v > v_z \\ \nu_{ez}(v_z), \quad v < v_z \end{array} \right] \right\}, \\ S_{ez1} &= -\frac{3v_z^2}{5v^2} \frac{\partial}{\partial v} \left\{ v^2 \frac{\partial f_0}{\partial v} \left[ \begin{array}{l} \frac{v_z}{v} \nu_{ez}(v), \quad v > v_z \\ \frac{v}{v_z} \nu_{ez}(v_z), \quad v < v_z \end{array} \right] \right\} \\ &+ \nu_z \frac{\partial f_0}{\partial v} \left[ \begin{array}{l} \nu_{ez}(v) \left( 1 - \frac{3}{5} \frac{v_z^2}{v^2} \right) \quad v > v_z \\ \frac{2}{5} \nu_{ez}(v_z), \quad v < v_z \end{array} \right] \\ \nu_{ez} &= \frac{4\pi e^2 Z^2 N_z \ln \Lambda}{m^2 v^3}, \quad S_{ezk} = \frac{2k+1}{2} \int_{-1}^1 S_{ez} P_k(\cos\theta) d(\cos\theta). \end{aligned} \quad (4)$$

Solving now by the usual method<sup>[3]</sup> Eq. (3) with the right-hand side of  $S_{ez}$  [Eq. (4)] known, we obtain the symmetrical  $f_0(v)$  and the directional  $f_1(v)$  parts of

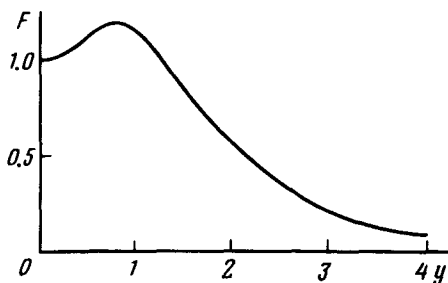


FIG. 1.

the distribution function, and consequently determine the electron heating and the average velocity of the electrons relative to the ions, i.e., the electron current. It can be represented in the form

$$j_e = -Z j_{z0} F(y), \quad j_{z0} = e N_z v_z = e N_{z0} v_0, \quad y = v_z (m/2T_e)^{1/2}. \quad (5)$$

The function  $F(y)$  is shown in Fig. 1.  $F(y) = 1 + O(y^2)$  as  $y \rightarrow 0$  and  $F = 4.80/y^3$  at  $y \gg 1$ . The maximum value  $F_m \approx 1.18$  is reached at  $y \approx 0.8$ . We present an analytic expression for  $F(y)$  obtained in second order in the expansion of the distribution function in Laguerre polynomials

$$F(y) = 1.950 F_0(y) - 0.5545 F_1(y) - 0.06295 F_2(y)$$

$$F_0(y) = \frac{3\pi^{1/2}}{4} \Phi(y) - \frac{3}{2} y e^{-y^2}, \quad F_1(y) = -\frac{9\pi^{1/2}}{4} \Phi(y) + \frac{9}{2} y e^{-y^2} (1 + y^2), \quad (6)$$

$$F_2(y) = \frac{15}{8} y^3 e^{-y^2} (1 - 2y^2).$$

Formula (6) is accurate to 1–2%.

In (5),  $j_{z0}$  is the density of the current of multiply charged ions. It is seen from (5) that the electron current produced in the plasma is approximately  $Z$  times larger than the  $Z$ -ion current.

The electron current (5) has a nonzero divergence, and should produce a strong electrostatic field in the plasma. However, under the conditions  $D \ll R_0$  and  $D \ll L$  ( $R_0$  is the beam radius and  $D$  is the Debye radius) the plasma is quasineutral,  $N_e \approx N_i$ . This means that the combined electric current, which is determined by the action of the  $Z$  ions and of the electrostatic field  $E = -\nabla\phi$ , has no divergence,  $\nabla(j_e + j_E) = 0$ . This leads to a quasineutrality equation that determines the potential of the electric field generated in the plasma<sup>[5]</sup>:

$$\nabla(\hat{\sigma} \nabla \phi) = \nabla j_e. \quad (7)$$

Here  $\hat{\sigma}$  is the plasma conductivity tensor and  $j_e$  is the source current (5). If we neglect the effect exerted on the conductivity by the magnetic field produced in the plasma as well as the change in the electron temperature, then  $\sigma$  is a constant and Eq. (7) is identical with the Poisson equation. Its solution is obtained by expansion in spherical functions. In the case of an axisymmetric ion

beam we have for the electric and magnetic fields:

$$\phi = \sum \phi_k(r) P_k(\mu), \quad H_\phi = \sum H_{\phi k}(r) P_k(\mu),$$

$$\phi_k(r) = -\frac{4\pi}{2k+1} \left\{ r^k \int_0^{R_0} t^{-k+1} \rho_k(t) dt + r^{-k-1} \int_0^r t^{k+2} \rho_k(t) dt \right\}, \quad (8)$$

$$H_{\phi k} = -\frac{4\pi r}{c k(k+1)} \left( \frac{d\phi_k}{dr} + j_{er k} \right), \quad \rho_k = \frac{2k+1}{8\pi} \int_{-1}^1 \left[ \frac{\partial j_e}{\partial x} + \frac{\partial j_e^-}{\partial x} \right]_{r,\theta} P_k(\mu) d\mu. \quad (9)$$

Here  $(r, \theta, \phi)$  is a spherical system with origin at the center of the ion beam on the plasma boundary  $x=0$  and with an angle  $\theta$  reckoned from the  $x$  axis,  $\mu = \cos\theta$ ,  $j_{er}$  is the radial component of the current  $j_e$  (5), and  $\partial j_e^- / \partial x$  is a fictitious source in the region  $x > 0$  (outside the plasma) resulting from the condition that the current must not flow through the plasma boundary  $x=0$  [ $j_e^-(x) = -j_e(x)$ ]. An essential role is played in formulas (9) by the discontinuities of the current  $j_e$ , which occur at the plasma boundary  $x=0$  and at the point where the  $Z$ -ion beam is stopped,  $\xi_k = L\Psi(y_0)$  [see (2) and Fig. 1]; actually, however, the discontinuity regions are smeared out over a distance on the order of  $l$ .

The total electron current forms according to (5), (8), and (9) a closed configuration. In the region occupied by the ion beam it is directed, just as the ion current, in the  $x > 0$  direction—the source current (5) predominate here. Outside this region, the electron current is oppositely directed and is produced by the electrostatic field. The current density increases sharply near the plasma boundary  $x \approx 0$  and  $x \approx x_k$  at the beam stopping point. The magnetic field of the combined current has only the component  $H_\phi$  [Eq. (8)]. It is annular in shape and encircles the ion beam. The maximum value is  $H_{\phi m} \sim \pi R_0 Z j_{e0} / c$ .

Estimates show that the current and the magnetic field produced in the plasma can be appreciable under real conditions.<sup>[6-8]</sup> The condition for the appearance of a strong current is  $l < R_0 < L$ .

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<sup>1)</sup>The phenomenon under consideration is similar in character to the dragging of multiply charged ions in a plasma by an electron flux (runaway of multiply charged ions), which is considered in<sup>[1,21]</sup>.

<sup>1</sup>A. V. Gurevich, Zh. Eksp. Teor. Fiz. 40, 1825 (1961) [Sov. Phys. JETP 13, 1282 (1961)].

<sup>2</sup>H. P. Furth and P. H. Rutherford, Phys. Rev. Lett. 28, 545 (1972).

<sup>3</sup>I. P. Shkarofsky, T. Johnston, and M. P. Bachynski, Particle Kinetics of Plasmas, Addison-Wesley, 1966 (Russ. transl., Atomizdat, 1969).

<sup>4</sup>A. D. Landau, Zh. Eksp. Teor. Fiz. 7, 203 (1977).

<sup>5</sup>A. V. Gurevich, A. K. Krylov, and R. E. Tsedeliha, Space Sci. Rev. 19, 59 (1976).

<sup>6</sup>C. L. Olson, Sandia Labor, Rep. SAND 76-0292, 1976.

<sup>7</sup>A. W. Maschke, 1975 Part Accel. Conf., Washington, D. C., March 12-14, 1976.

<sup>8</sup>R. N. Sudan, Cornell Univ. Report LPS 200, 1976.