Charge distribution in anomalous nuclei

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The effect of fermion-charge accumulation on the conditions for the existence anomalous superheavy nuclei and of their properties is investigated.

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Much interest has been shown recently in the possible existence of anomalous nuclei, both as a result of the appearance of concrete theoretical models,[1–3] and in connection with the first experimental attempts at observing such nuclei.[4–6] The theory of pion condensation[3] predicts the existence of superdense anomalous nuclei of two types; nuclei with mass numbers $A < A_1 \sim 10^2$—10^3 and $\nu = Z/A \approx 0.5$, and also superdense superheavy nuclei with $A > A_2 \sim 10^4$—10^5 and $\nu \ll 1$. In the latter, the observable charge Z is large $(Ze^2/R \sim m_\pi c^2)$ and, as follows from the results of[7] fermions should accumulate in them.

The accumulation of fermions is due not only to the production of e^+e^- and $\mu^+\mu^-$ pairs from vacuum,[7] but also to the β processes $n \longleftrightarrow p + e^- + \tilde{\nu}_e$, $p \to n + e^+ + \nu_e$ and $n \longleftrightarrow p + \mu^- + \tilde{\nu}_\mu$, $p \to n + \mu^+ + \nu_\mu$. The conditions for the equilibrium of the system with respect to these processes and the reaction $n \to p + \pi^-$ were obtained in an approximation in which the system was assumed to be quasihomogeneous (the potential varies slowly over the characteristic length). The density of the electrons and of the μ^- mesons, in the Thomas-Fermi approximation generalized to include the relativsitic case,[7] is equal to ($\hbar = c = 1$ and the potential is measured in energy units)

$$n_{e,\mu} = (3\pi^2)^{-1}[(\epsilon_{max} + V(r))^2 - m^2_{e,\mu}]^{3/2}\,\theta(V(r) + \epsilon_{max} - m_{e,\mu}), \tag{1}$$

where ϵ_{max} is the energy below which all the electron and μ^--meson levels are filled ($|\epsilon_{max}| < m_e$), and is equal to the difference between the chemical potentials of the neutrons and protons. To find the charge density of the hadron subsystem (baryons + pions) we used, as in[3], an exactly solvable model of the limiting condensate field,[8] but, in contrast to[3], we did not confine ourselves to a homogeneous distribution of the charge over the volume of the nucleus:

$$n_h(r) = (n/2 - F^2 V(r)/4)\theta(R - r). \tag{2}$$

The first term is the charge density of the baryon quasiparticles (n = const), and the second is the density of the pion charge; $F = 1.35 m_\pi$ is the pion decay constant. As a result, the potential is determined from the equation

$$\Delta V = 4\pi e^2 [\, n_e(r) + n_{\mu^-}(r) - n_h(r)\,]\ . \tag{3}$$

This equation was solved analytically for two limiting cases, $A \gg 1/e^3$, and $A \ll 1/e^3$, and numerically in the intermediate region $A \sim 1/e^3$. We shall consider the first effect, since the effect of fermion accumulation is significant precisely in this mass-number region.

In this case the potential is practically constant within the nucleus and changes only near its surface, over distances much shorter than the nuclear radius R. This allows us to neglect the curvature of the edge of the nucleus ($\Delta V \approx V''$), thereby greatly simplifying the problem.

Inside the nucleus, the charge of the baryon subsystem can be screened either by fermions or by condensate pions. At $n \ll n^* \approx (\sqrt{3/2})(\pi/2)F^3 \approx 10 n_0$, pion screening is energywise favored, and the screening fermion charge accumulates outside the nucleus. In the opposite limiting case ($n \gg n^*$) fermion screening is realized both inside and outside the nucleus. In actual practice the intermediate case takes place.

In these limiting cases, the solution of (3) can be written with sufficient accuracy (see[7]) in the form

$$V(r) = \begin{cases} V_0 (1 - c e^{\mu x}) & r < R \\ V_0 \beta/(x + b) & r < R \end{cases}, \tag{4}$$

where $x = (r - R)/\lambda$, $\lambda = \sqrt{3}\,\pi/2eV_0$, and V_0 is the value of the potential inside the nucleus and is determined from the electroneutrality condition as $x \to -\infty$. The constants b and c are obtained by matching the values of V and V' on the nuclear boundary:

$$b = \left(\frac{\beta}{2} + \sqrt{\frac{\beta^2}{4} + \frac{\beta}{\mu}}\right) \sim 1, \qquad c = 1 + \frac{\mu\beta}{2} - \sqrt{\frac{\mu^2\beta^2}{4} + \mu\beta}. \quad (5) \tag{5}$$

We present the values of V_0, λ, μ, and β;

$$n \gg n^*: \quad V_0 = (3\pi^2 n/4)^{1/3}, \quad \lambda = \left(\frac{3}{\pi}\right)^{1/6} e^{-1}(2n)^{-1/3},$$

$$\mu = \left[6 + \left(\frac{\pi^2 F^2}{2} + m_\mu^2\right)\frac{3}{2V_0^2}\right]^{1/2} \sim 1, \qquad \beta = 1, \tag{6}$$

$$n \ll n^*: \quad V_0 = 2n/F^2, \quad \lambda = \sqrt{3\pi}F^2/4en, \quad \mu = \sqrt{3 + \frac{3\pi^2 F^2}{4V_0^2}}, \qquad \beta = \sqrt{2}$$

The limiting-screening approximation is valid when $R/\lambda \gg 1$, which yields $A \gg 1/e^3$ for the case $n \gg n^*$ (fermion screening) and $A \gg (1/e^3)(n^*/n)^2$ in the opposite limiting case (pion screening).

Thus, the internal part of a superheavy superdense nucleus comprises an electrically neutral plasma consisting of baryon quasiparticles, pions, electrons, and μ^- mesons. The potential inside the nucleus is constant and equal to V_0. The electric field and the surface charge are concentrated in a transition layer of width λ. The field intensity $\mathcal{E}$ reaches a maximum value at the edge of the nucleus: $\mathcal{E}_{max} = V'(x=0)/\lambda e \gtrsim \mathcal{E}_0$, where $\mathcal{E}_0 = \pi_\pi^2 c^3/e\hbar \approx 10^{21}$ V/cm. The total charge Z' inside the nucleus is equal to $Z' = ZV_x'(0)R^2/\lambda e^2 \sim Z(Ze^3)^{1/3}$. In a layer of width $\lambda \ll \Delta r \ll R$ near the nucleus is located a charge that cancels out Z' accurate to terms $\sim A^{1/3}$, i.e., the observable charge of the nucleus Z_1 increases like $A^{1/3}$. To calculate Z_1 exactly when A and n are specified, it is necessary to match together, at the point where the charge density vanishes ($V = m_e - \epsilon_{max}$), the solution of Eq. (3) and the Coulomb potential $Z_1 e^2/R$. The answer depends on the level ϵ_{max} up to which the fermion levels are filled ($Z_1 = 0$ at $\epsilon_{max} = m_e$ and Z_1 is maximal at $\epsilon_{max} = -m_e$). Figure 1 shows the results of a numerical calculation of Z_1. It is seen that Z_1 varies linearly with $A^{1/3}$ as $A \to \infty$; the proportionality coefficient is in this case smaller by one order of m magnitude than the value obtained in[3] without allowance for the fermion accumulation.

The production of an electrically neutral plasma in the interior of the nucleus and the accumulation of the fermion charge outside the nucleus decrease greatly the Coulomb energy that makes heavy nuclei unstable to fission. In the case of maximum screening, the electrostatic energy reduces to the surface energy $E_Q = -\int V\Delta V\, d^3r/8\pi e^2 \sim A/(Ae^3)^{1/3}$. In this case ($A \gg 1/e^3$) the system energy does not have any terms that increase more rapidly than A with increasing A, i.e., the superheavy nuclei are stable to fission at $A > A_2 \sim 1/e^3$.

As a result of fermion accumulation, the internal part of a superheavy nucleus is not pure neutron matter with π condensate, but neutron-star matter. It is well known that the latter has a lower energy. This increases, compared with[3], the range of nuclear-constant values at which anomalous superheavy nuclei can exist. The fact that the region of fermion screening is located at large densities leads to an increase of the equilibrium density by a factor $(2-3)n_0$.

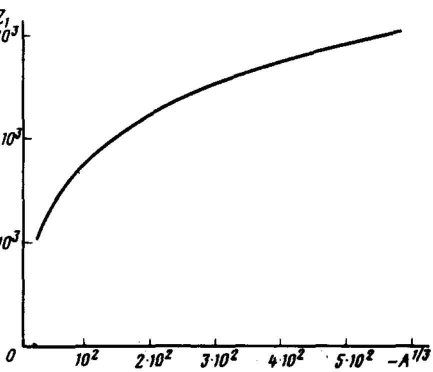

FIG. 1. Observed charge Z_1 of superheavy nucleus as a function of $A^{1/3}$. The calculation pertains to the case $n = 7n_0$ and $\epsilon_{max} = -m_e$ (maximum observable charge).

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