Nonlinear localized magnetization wave of a ferromagnet as a bound state of a large number of magnons

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A new type of localized two-parameter solutions of the one-dimensional dynamic equations for the magnetization in a ferromagnet is obtained. The energy E of the nonlinear wave is calculated as a function of its momentum P and of the number of bound magnons in the wave. A formally periodic dependence of E on P having the meaning of the dispersion law for a magnetic soliton, is observed.

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In a quantum analysis of spin oscillations in a one-dimensional ferromagnetic chain, Bethe[1] has observed and described the so-called spin complexes. Recently, by macroscopic analysis of the dynamics of a one-dimensional ferromagnet, various localized solutions were obtained for the nonlinear equations for the magnetization.[2–4] These solutions can be treated as bound states of a large number of spin waves. It is of interest to establish the connection between the classical localized states and the spin complexes. However, the solutions obtained in[2–5], do not permit a direct comparison with the results of the quantum calculation.

We have obtained a two-parameter localized solution of the nonlinear equations for the magnetization of a ferromagnet; this solution permits, after quasiclassical quantization, a comparison with Bethe's results.[1] We describe this solution in this paper and discuss the feasibility, in principle, of experimentally observing localized waves in a ferromagnet.

We shall consider a ferromagnet with an anisotropy of the easy-axis type and describe its state with the aid of a magnetization vector M. Using the condition $M^2=M_0^2$, which is natural for a ferromagnet, we write down the components of M in the form

$$M_x = M_0 \sin\theta\cos\phi, \quad M_y = M_0 \sin\theta\sin\phi, \quad M_z = M_0\cos\theta,$$

where $M_0=2\mu_0 s/a^3$ (μ_0 is the Bohr magneton, s is the spin of the atom, and a^3 is the volume of the unit cell).

The dynamic equations for the vector M (the Landau-Lifshitz equations without relaxation) can be regarded in terms of the angle variables θ and ϕ as the Euler equations corresponding to the following Lagrange functions[6]

$$L = \frac{\hbar M_0}{2\mu_0}(1-\cos\theta)\frac{\partial\phi}{\partial t} - W\{\theta,\phi\},$$

where W is the density of the magnetic energy:

$$W = \frac{a}{2}M_0^2\{(\nabla\theta)^2 + \sin^2\theta(\nabla\phi)^2\} + \frac{1}{2}\beta M_0^2\sin^2\theta.$$

Here α is the exchange constant, β is the anisotropy constant ($\beta > 0$), and the z axis is chosen along the easy axis.

We study a localized plane magnetization wave, in which the fields θ and ϕ depend on a single spatial coordinate ξ and on the time t (one-dimensional solution):

$$\theta = \theta(\xi - Vt), \qquad \phi = \psi(\xi - Vt) + \omega t,$$

where V is the displacement velocity of the localized wave and ω is the precession frequency in a reference frame moving with velocity V. It is required that the function $\theta(\xi)$ vanish, and that the derivative $d\psi/d\xi$ be bounded at infinity.

Before writing down the solution, we note that it is characterized by two parameters, ω and V. It is more convenient, however, to use other parameters, namely the integrals of motion N and P:

$$N = \frac{M_0 a^2}{2\mu_0}\int_{-\infty}^{\infty}(1-\cos\theta)\,d\xi, \qquad P = -a^2\int_{-\infty}^{\infty}\frac{\partial\phi}{\partial\xi}\,\frac{\partial L}{\partial(\partial\phi/\partial t)}\,d\xi,$$

where N is the number of spin deviations (magnons) and P is the momentum of the magnetization field.

The simplest forms of the functions $\theta(\xi)$ and $\psi(\xi)$ are determined by the relations

$$\mathrm{tg}^2\left(\frac{\theta}{2}\right) = \frac{\mathrm{sh}^2\left(\frac{N}{N_1}\right) + \sin^2\left(\frac{\pi P}{2P_0}\right)}{\mathrm{ch}^2(\kappa\xi) - \sin^2\left(\frac{\pi P}{2P_0}\right)}, \qquad \frac{d\psi}{d\xi} = -\frac{\beta V}{2a\omega_0}\,\frac{1}{\cos^2(\theta/2)}\,. \tag{1}$$

Here

$$N_1 = \frac{2a^2 M_0}{\mu_0}\sqrt{a/\beta} = \frac{4s}{a}\sqrt{a/\beta}, \quad P_0 = \frac{\pi\hbar a^2 M_0}{\mu_0} = \frac{2\pi s\hbar}{a},$$
$$\hbar\omega_0 = 2\beta\mu_0 M_0 .$$

It follows from (1) that the solution for θ is localized in the region $\Delta\xi = 1/\kappa$, and

$$\kappa(P, N) = \sqrt{\frac{\beta}{a}}\,\mathrm{th}\left(\frac{N}{N_1}\right)\left\{1 + \frac{\sin^2\left(\frac{\pi P}{2P_0}\right)}{\mathrm{sh}^2\left(\frac{N}{N_1}\right)}\right\}. \tag{2}$$

If we express κ in terms of V and ω and impose the obvious requirement that κ^2 be positive, then we obtain a condition for the existence of localized solutions (1):

$$\frac{a}{\beta}\kappa^2(V,\omega) = 1 - \frac{\omega}{\omega_0} - \frac{\beta}{a}\left(\frac{V}{2\omega_0}\right)^2 > 0\,. \tag{3}$$

At $\omega = 0$, the solution (1) describes a solitary spin wave,[2,4] which can move only with velocity $V < 2\omega_0\sqrt{\alpha/\beta}$. At $V = 0$ and $\omega > 0$, the solution (1) corresponds to an immobile self-localized magnetization state.[5] Finally, at $\kappa^2 \ll \beta/\alpha$ we obtain

$$\theta = \theta_0\,\mathrm{sech}[\kappa(\xi - Vt)], \tag{4}$$

where

$$\theta_0 = \kappa\sqrt{\frac{\alpha}{\beta}}\left\{1 + \frac{\beta}{\alpha}\left(\frac{V}{2\omega_0}\right)^2\right\}^{-1/2}. \tag{5}$$

The solution (4) coincides with the principal approximation of the asymptotic expansion in[3] and describes a magnetic soliton whose velocity is actually determined by the condition $\kappa(V,\omega)=0$.

The magnetization-field energy given by solution (1) is equal to

$$E(P,N) \equiv a^2\int_{-\infty}^{\infty} W d\xi = 4a^2 M_0 \kappa(P,N), \tag{6}$$

where $\kappa(P,N)$ is determined by formula (2).

It turns out that

$$V = \partial E/\partial P, \qquad \hbar\omega = \partial E/\partial N.$$

It is easy to verify that the energy of the localized wave per magnon satisfies the condition

$$\epsilon(P,N) = E(P,N)/N < \hbar\omega(k), \qquad P = Nk, \tag{7}$$

where $\omega(k)=\omega_0[1+(\alpha/\beta)k^2]$ gives the long-wave law of magnon dispersion. It follows from (7) that the localized magnetization wave (1) corresponds to a bound state of N magnons.

At $\beta=0$ and $s=\frac{1}{2}$ we readily obtain from (5) the expression

$$E(P,N) = (2I/N)\sin^2(aP/2\hbar), \tag{8}$$

where I has the meaning of an exchange integral expressed in terms of the macroscopic parameters by the relation $Ia^2=4\alpha\mu_0 M_0$. The energy (8) coincides exactly with Bethe's result.[1]

A curious feature of formulas (6) and (8) is the presence of a periodic dependence of E on P, with a period $2P_0$. The momentum of the waves is transformed into a quasimomentum. It turns out that a one-dimensional ferromagnet possesses, as it were, a certain structure that manifests itself when a spin complex propagates in the magnet. This structure is characterized by a linear dimension $\Delta X=\mu_0/M_1$ ($M_1=a^2M_0$ is the nominal magnetic moment per unit length of the magnet), which determines the maximum density of the number of flip spins along the one-dimensional magnet.

It must be borne in mind that the noted periodicity is formal in a certain sense, since the long-wave approximation in the description of the ferromagnet turns out to be valid only at $|P|<P_0$. At $N\gg N_1$, however, our formulas remain valid all the way to small vicinities of the points $P=\pm P_0$.

The solitary magnetization waves can probably be observed by the same procedure that we used in the study of nonlinear thermal pulses.[7] The possibility of its realization in magnetic measurements has apparently been confirmed by experiments[8] on the observation of individual moving spin excitations. If the amplitude of the nonlinear wave is not very high, then the wave takes the form of a soliton (4), the characteristics of which are connected by relation (5). The presence of internal precesion can be detected by using the relation $\kappa(V,\omega)=0$, from which it follows that the connection between ω and V is practically the same as the connection between the frequency and the group velocity of single-magnon excitation.

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