

Asymptotic form of hadronic form factors in the quark model

V. L. Chernyak and A. R. Zhitniskii

Irkutsk State University

(Submitted April 7, 1977)

Pis'ma Zh. Eksp. Teor. Fiz. **25**, No. 11, 544-547 (5 June 1977)

The dependence of the asymptotic hadronic form factors on the quantum numbers of the hadrons is obtained. Asymptotic forms of mesic and baryonic form factors of different tones are calculated.

PACS numbers: 12.40.Cc, 13.40.Fn

A connection between the asymptotic forms of the hadronic form factors and the quark content of the hadrons was noted in^[1,2]. In the present article we elucidate the dependence of the asymptotic form factors on the quantum numbers of the target.

In the spirit of the models that make use of the expansion of operators on the light cone, we take interaction at small distances into account only in the lowest order in the coupling constant,¹⁾ whereas the matrix elements of the operators are assumed to be exactly calculated. For mesons, in this approach, the form factor is expressed as a sum of the contributions of four diagrams of the type shown in Fig. 1, in which the external quark lines are operators. Thus, the diagram of Fig. 1 is proportional (D and S are the gluon and quark free propagators and Γ is the external-current vertex):

$$\int dx dy \langle M_2 | \bar{\psi}_\alpha(x) \psi_\beta(y) | 0 \rangle (\gamma_\lambda S(x) \Gamma)_{\alpha\tau} (\gamma_\rho)_{\sigma\beta} D_{\lambda\rho}(x-y) \langle 0 | \bar{\psi}_\sigma(y) \psi_\tau(0) | M_1 \rangle. \quad (1)$$

We describe here only the scheme for calculating (1). It is convenient to carry out the expansion

$$16 \bar{\psi}_\alpha \psi_\beta = 4 \delta_{\alpha\beta} (\bar{\psi} \psi) + (\gamma_\mu)_{\beta\alpha} (\bar{\psi} \gamma_\mu \psi) + \dots$$

For the matrix element of each of the bilocal operators²⁾ we choose the most general form, for example

$$\langle p J \lambda | \bar{\psi}(x) \gamma_\mu \psi(y) | 0 \rangle = \exp i p(x+y)/2 \{ p_\mu e^{* \lambda}_{\mu_1 \dots \mu_y} z_{\mu_1 \dots z_{\mu_y}} \phi_{1\nu}^J(zp) + \dots \},$$

where $z = x - y$ and e^λ is the meson polarization vector. We set z^2 in the functions ϕ_i^J equal to zero, so that $z^2 \sim x^2 \sim y^2 \sim 1/q^2$. Lastly, introducing the Fourier transforms $\phi_i^J(zp) = \int d\xi \phi_i^J(\xi) \exp i \xi(zp)$ and integrating in (1) with respect to x and y we obtain the final answer. In the c. m. s. of the mesons, the answer for the vector (axial) current takes the form³⁾ $(\lambda_1 + \lambda_2 = \lambda_J, n = 2$ for

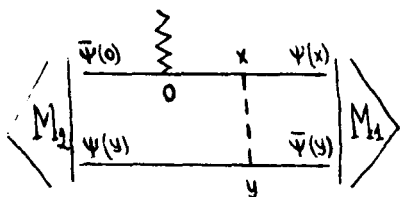


FIG. 1.

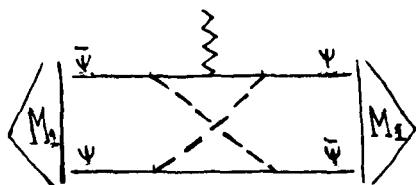


FIG. 2.

mesons):

$$\langle p_2 J_2 \lambda_2 | V_{\lambda J}(A) | p_1 J_1 \lambda_1 \rangle \sim (1/\sqrt{q^2})^{|\lambda_2 - \lambda_1| + 2n - 3}. \quad (2)$$

For the tensor (scalar, pseudoscalar) current:

$$\langle p_2 J_2 \lambda_2 | T_{\lambda J}(S, P) | p_1 J_1 \lambda_1 \rangle \sim (1/\sqrt{q^2})^{|\lambda_2 - \lambda_1| + 2n - 4} \quad (3)$$

[with the exception of the case $\lambda_1 = \lambda_2 = 0$, when $\langle \lambda_2 = 0 | T_{\lambda=0}(S, P) | \lambda_1 = 0 \rangle \sim 1/q^2$]. The decrease of the asymptotic values with the increasing helicity in (2) and (3) is due mainly to the fact that $e_p^\pm \sim \text{const}$, whereas $e_p^0 \sim p_p/M$.

We present now a simple and illustrative interpretation of the results (2) and (3) for the case of a free quark-antiquark in a state with the required set of quantum numbers, inasmuch as the results of (2) and (3) hold in this case. Consider, for example, the diagram of Fig. 3, which is calculated in accordance with the usual rules. In the c.m.s. of the mesons, the quarks have small transverse momenta (\mathbf{p}_\perp and \mathbf{q}_\perp) and are located on (or close to) the mass shell. Such a diagram does not correspond to the form factor of any definite meson, since the initial and final states are products of plane waves (i.e., $|1\rangle = |p_1 \lambda_1, p_2, \lambda_2\rangle$). In other words, the initial and final states on the diagram 2 have neither a definite total angular momentum J_1 (J_2), nor its projections J_{z1} (J_{z2}) on the z axis (the direction of motion). The mesic state can be constructed in the following manner: a) we form first, in the meson rest system, the state

$$|J M \lambda_1 \lambda_2\rangle = \int d\phi d\cos\theta D_{M\lambda}^J(\theta, \phi) |p, \theta, \phi, \lambda_1, \lambda_2\rangle;$$

b) taking a linear combination with different $\lambda_{1,2}$, we form a state with definite P -parity, C -parity, etc.; c) we perform a boost along (or opposite to) the z axis. Thus, the mesic form factor can be obtained from diagram 2 by multiplication by appropriate D^J functions and by integration over the angles $\theta_{1,2}$ and $\phi_{1,2}$. The diagram of Fig. 2 has a nontrivial dependence on $\cos\theta_1$ and $\cos\theta_2$, so that integration with respect to $\phi_{1,2}$ does not influence the asymptotic form. In other words, the asymptotic form does not depend on the meson spins. At the same time, the dependence on the angles $\phi_{1,2}$ enters in the diagram of Fig. 2 only via the spinors ($\sim \exp \pm i\phi/2$) or via $(\mathbf{p}_\perp \cdot \mathbf{q}_\perp/q^2)^k$ (in the expansion of the gluon propagator).⁴⁾ Therefore each unit of L_z (of the projection of the angular

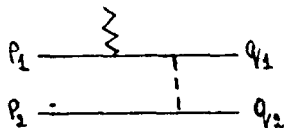


FIG. 3.

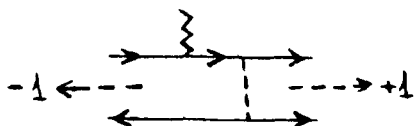


FIG. 4.

momentum of the quarks) leads to suppression of the asymptotic form by a factor $(1/\sqrt{q^2})$.⁵⁾ Let us examine, finally, the form factor $\langle \lambda_2 = 2 | V_{\lambda=1} | \lambda_1 = -1 \rangle$. Its asymptotic form is determined by the diagram of Fig. 4 (the arrows on the quark lines indicate the helicities of the quarks). Compared with the "normal" behavior of the form factor ($\sim 1/\sqrt{q^2}$), there is an additional suppression by a factor $(1/\sqrt{q^2})^2$ on account of $L_{z_1} = -1$, $L_{z_2} = 1$, and also by a factor $(1/\sqrt{q^2})$, because of the reversal of the helicity of the quark. On the whole we have $\langle \lambda_2 = 2 | V_{\lambda=1} | \lambda_1 = -1 \rangle \sim 1/q^4$, which agrees with (2).⁶⁾

All the arguments advanced above are valid also for baryon form factors. The answers given by formulas (2) and (3) with $n=3$ [with the exception of $\langle \lambda_2 = \frac{1}{2} | T_{\lambda=1} | \lambda_1 = \frac{1}{2} \rangle \sim 1/q^4$]. In particular, we predict that all the off-diagonal $\gamma N \rightarrow N^* \gamma$ vertices will have, after averaging over the polarizations, the same asymptotic forms as the elastic $\gamma N \rightarrow N$ vertex.

We note in conclusion that our results do not agree with those of Brodsky and Farrar,^[2] who have predicted an additional suppression $\sim (1/q^2)^l$ (l is the orbital angular momentum of the quarks in the meson rest system).

We are deeply grateful to A. I. Vainshtein for useful discussions.

¹⁾Interaction at short distances (e.g., the diagram of Fig. 2 and others), will be accounted for in a separate paper.

²⁾To maintain an explicit gauge invariance, we can make the substitution $\bar{\psi}(x)\psi(y) \rightarrow \bar{\psi}(x) \exp i g \int_y^x d\sigma_\mu \beta_\mu(\sigma) \psi(y)$, (see^[3]).

³⁾We assume that $\Phi_i^j(\xi) \rightarrow 0$ as $|\xi| \rightarrow 1$ rapidly enough to keep this region from influencing the asymptotic form. This condition is satisfied for three quarks (see below).

⁴⁾If $|\lambda_y| = 1$, then $(p_{1y}/\sqrt{q^2})$ or $(q_{1y}/\sqrt{q^2})$ can appear once in the numerator. All these properties follow in fact from the Lorentz invariance and therefore remain valid also when account is taken of higher orders of perturbation theory.

⁵⁾This statement is valid for quarks on the mass shell with mass $m \neq 0$. If we imitate the bound state by introducing the distributions $\int dp_{1,2}^2 \rho(p_{1,2}^2)$ with respect to $p_{1,2}^2$ and $q_{1,2}^2$, then the asymptotic forms can be subject to the influence of the region $(1 \pm \cos \theta_1 + p_{1,2}^2/\mu^2)(1 \pm \cos \theta_2 + q_{1,2}^2/\mu^2) |q^2| \sim \mu^2$, $p_{1,2}^2 \rightarrow 0$, $|\cos \theta_{1,2}| \rightarrow 1$, in which the gluon propagator becomes "light" ($\sim 1/\mu^2$, $\mu^2 \sim 1 \text{ GeV}^2$). It is easy to verify that this region will not influence the asymptotic form at $\int dp_{1,2}^2 \rho(p_{1,2}^2) \rightarrow 0$ as $p_{1,2}^2 \rightarrow 0$. The equivalent condition in terms of the Feynman distributions $\int_0^1 dx F(x)$ is $[F(x)/\sqrt{1-x}] \rightarrow 0$ as $x \rightarrow 1$.

⁶⁾In the case of free quarks there is an additional [compared with (3)] suppression ($\sim 1/q^2$) in the matrix elements of the S , P , and T currents for mesons with quantum numbers $[P = -C = (-1)^{J-1}]$ at $|\lambda_{1,2}| > 1$.

- ¹V. Matveev, R. Muradyan, and A. Tavkhelidze, *Nuovo Cimento Lett.* **7**, 719 (1973).
- ²S. Brodsky and G. Farrar, *Phys. Rev.* **D11**, 1309 (1975).
- ³D. Gross and S. Treiman, *Phys. Rev.* **D4**, 1059 (1971).