Manifestation of the proximity of atomic nuclei to the point of the π-condensate instability in inelastic scattering of nucleons

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It is shown that the behavior of the differential cross sections of inelastic scattering of nucleons by even—even nuclei accompanied by excitation of states with anomalous parity in the region where the transferred momenta k are of the order of the Fermi momentum p_F is a characteristic of the degree of proximity of nuclei to the point of the π-condensate instability.

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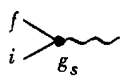
The question of π condensation in atomic nuclei[1] has aroused great interest. An analysis[2] of the spectroscopic data and of the interaction of the slow pions with nuclei offer evidence that there is no π condensate in atomic nuclei, but they are close to the point of the π-condensate instability. It is very important to determine the degree of this proximity in connection with the possible existence of anomalous nuclei.[3] We shall show that nuclear reactions can serve as a sensitive test of this proximity.

We consider the reaction of inealstic scattering of a nucleon by an even-even nucleus with excitation of a state of anomalous parity $(0^-, 1^+, 2^-, \cdots)$. Within the framework of the distorted-wave method, the amplitude of this process is determined by the matrix element

$$M_{if} =$$

$$= (\Psi_f, g_s \Psi_i), \qquad (1)$$

where Ψ_i and Ψ_f are the wave functions of the problem of nucleon scattering by an optical potential. The amplitude g_s for the production of the state $|s\rangle$ is determined by a homogeneous equation in the form[4]

$$g_s = F A g_s, \qquad (2)$$

where F is the quasiparticle-interaction amplitude and is irreducible in the particle-hole channel,

$$A(\mathbf{r}, \mathbf{r}'; \omega_s) = \sum_{\lambda\lambda'} \frac{n_\lambda - n_{\lambda'}}{\epsilon_\lambda - \epsilon_{\lambda'} - \omega_s} \phi_\lambda(\mathbf{r}) \phi_\lambda^*(\mathbf{r}') \phi_{\lambda'}(\mathbf{r}') \phi_\lambda(\mathbf{r}). \qquad (3)$$

n_λ, ϵ_λ, and ϕ_λ are the occupation numbers, the energies, and the wave functions of the quasiparticles, and ω_s is the excitation energy. For states of anomalous parity, there remain in (2) only the spin-dependent terms F, including the amplitude of one-pion exchange in the annihilation channel.[5] The explicit form of F and of Eq. (2) in the coordinate representation, as well as a description of the method of determining $g_s(r)$ from (2) in the coordinate representation, can be found in[6].

For approximate estimates we can regard Ψ_i and Ψ_f in (1) as plane waves. Then $M_{if}=\int \exp(i\mathbf{kr})g_s(\mathbf{r})\,d\mathbf{r}\equiv g_s(\mathbf{k})$. We shall show that if the interaction parameters are close to critical, then the form factor $g_s(\mathbf{k})$ has a sharp maximum at $k\sim k_0$, where k_0 is the momentum at which the π-condensate instability occurs in nuclear matter, with $k_0\sim p_F$.[1] This is easiest to demonstrate for low-lying states of anomalous parity, which, as a rule, have low collectivity: their energies ω_s are shifted little relative to the nearest single-particle difference $\omega_0=\omega_2-\omega_1$, and in the transition density $\rho_s\equiv Ag_s=\sum_{\lambda\lambda'}\rho^s_{\lambda\lambda'}\phi_\lambda\phi^*_{\lambda'}$ of these states the most predominant of the coefficients $\rho^s_{\lambda\lambda'}$ is ρ^s_{12}. The remaining coefficients are small, but at $k\sim k_0$ their contribution turns out to be coherent and it is precisely this coefficient which is responsible for the resonant behavior of $g_s(\mathbf{k})$.[1)]

Expressing (3) in the form $A=A_0+A'$, where A_0 is the term corresponding to the transfer of the quasiparticles from the state $|\lambda\rangle=|1\rangle$ to the state $|\lambda'\rangle=|2\rangle$, we obtain from (2)

$$g=\Gamma' A_0 g, \tag{4}$$

where

$$\Gamma'=F+FA'\Gamma'. \tag{5}$$

In the momentum representation, introducing the wave function $\chi_0(\mathbf{k})=\int\exp(i\mathbf{kr})\times\phi_1^*(\mathbf{r})\phi_2(\mathbf{r})\,d\mathbf{r}$, of the separated particle-hole pair, we obtain from (4)

$$g_s(\mathbf{k})=-\frac{1}{\omega_0-\omega_s}\int\Gamma'(\mathbf{k},\mathbf{k}')\,\chi_0(\mathbf{k}')g_s(\mathbf{k}')\,d\mathbf{k}'. \tag{6}$$

At $k\sim p_F$ we can put[5] $\Gamma'(\mathbf{k},\mathbf{k}')=\Gamma(\mathbf{k})\delta(\mathbf{k}-\mathbf{k}')$, and we then obtain from (6)

$$g_s(\mathbf{k})\sim\Gamma(\mathbf{k})\chi_0(\mathbf{k}). \tag{7}$$

The quantity $\Gamma(\mathbf{k})$ is the total quasiparticle-interaction amplitude in infinite matter; it can be expressed in the form

$$\Gamma(\mathbf{k})=(G(\mathbf{k})+G'(\mathbf{k})\vec{\tau}_1\vec{\tau}_2)\vec{\sigma}_1\vec{\sigma}_2+T(\mathbf{k})(\vec{\sigma}_1\mathbf{k})(\vec{\sigma}_2\mathbf{k})\vec{\tau}_1\vec{\tau}_2, \tag{8}$$

where

$$G(\mathbf{k})=\frac{g}{1+2\Phi(\mathbf{k})g},\qquad G'(\mathbf{k})=\frac{g}{1+2\Phi(\mathbf{k})g'},$$

$$T(\mathbf{k})=\frac{t(k^2)}{[1+\Phi(\mathbf{k})(g'+t(k^2)k^2)]\,(1+2\Phi(\mathbf{k})g')}.$$

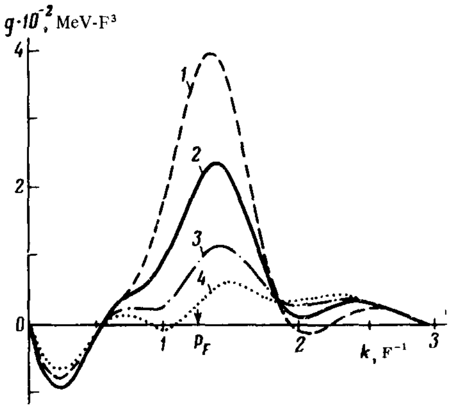


FIG. 1. Form factors for the excitation of the first 0^- level of the ^{208}Pb nucleus in the reaction (p,p') at different values of the spin-isospin interaction constant (see the text).

All the symbols are defined in[5]. The appearance of π-condensate instability is due to the vanishing, at $k=k_0 \sim p_F$ of the expression in the square brackets and the denominator of $T(\mathbf{k})$. Near the instability point, at $g'-g'_{cr} \ll 1$, this denominator can be represented in the form $c(k^2-k_0^2)^2+\omega_{min}^2$, where c is a number of the order of unity and ω_{min}^2 is a small quantity $[\omega_{min}^2 \sim \epsilon_F^2(g'-g'_{cr})$; ϵ_F is the Fermi energy and g'_{cr} is the critical value of the constant g' of the spin-isospin interaction]. As a result $T(\mathbf{k})$, and with it also $g_s(\mathbf{k})$ in (7), has a resonant behavior at $k \sim k_0$ if $\chi_0(\mathbf{k})$ in this region does not vanish for accidental reasons. Taking into account the spin-isospin variables, the form factor of the excitation of the state $|s\rangle$, having an angular momentum J, is given by the expression $g_s^i(\mathbf{k}) = g_s^{i(-)}(k)[Y_{J-1}\otimes\sigma]_J + g_s^{i(+)}(k)[Y_{J+1}\otimes\sigma]_J$. The superscript i labels the neutral excitations [reactions (p,p') and (n,n')] and the charged [(p,n) and (n,p)] excitations. For the 0^- level there remains only the component $g_s^{(+)}$.

Figure 1 shows a set of form factors for the first 0^- level in ^{208}Pb, excited in the reaction (p,p'). They were obtained by exact solution of Eq. (2) at realistic values of the parameters $\zeta_s=0.05$, $\alpha=0$, and $g'_{cr}=0.70$. Curve 2 corresponds to the value $g'=0.86$, which accounts for the experimental position of the 0^- level. Curve 1 corresponds to $g'=0.82$, a value closer to the instability point, curve 3 to $g'=0.90$, and curve 4 to $g'=0.98$. We see that the form factor at $k \sim p_F$ is indeed very sensitive to the extent to which the interaction parameters are close to critical.

An analogous conclusion holds also for other states of anomalous parity. At $J^\pi \neq 0^-$, the form factor $g_s(\mathbf{k})$ incorporates both components $g_s^{(-)}$ and $g_s^{(+)}$, which

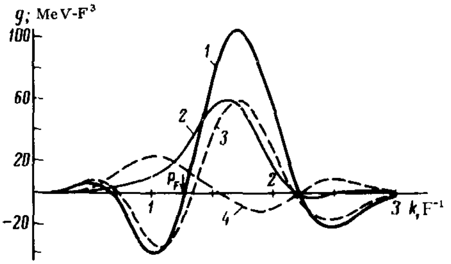


FIG. 2. Nuclear form factors for the reaction ^{208}Pb$(p,n)^{208}$Bi (transition $0^+ \to 5^+$). Solid curves 1 and 2—form factors $g^{(-)}$ and $g^{(+)}$, corresponding to orbital-momentum transfers $L=4$ and $L=6$, respectively at a spin-isospin interaction constant $g'=0.87$; dashed curves 3 and 4—the same form factors, respectively, calculated at $g'=1.02$.

contain the resonant term $T(k)$ with different weights. By way of example, Fig. 2 shows these components for the reaction (p,n) on the ^{208}Pb nucleus with transition to the ground state 5^+ of the ^{208}Bi nucleus. The picture here is more complicated, since the resonant dependence in (7) is strongly modulated by the function $\chi_0(\mathbf{k})$, which reverses sign near p_F [in contrast to the preceding case, where $\chi_0(\mathbf{k})$ has no zeros in the region p_F].

In the Born approximation with plane waves, the differential cross section is $d\sigma/d\Omega \sim (2J+1)(|g_s^{(-)}(k)|^2 + |g_s^{(+)}(k)|^2)$ and, as follows from the foregoing analysis, should have a maximum at angles corresponding to a momentum transfer $k \sim p_F$, and from the value of this maximum one can asses the degree of proximity of the nucleus to the π-condensate instability point. In a calculation with distorted waves, the resonant behavior of the cross section as a function of k becomes somewhat smoothed out, but this effect still remains. A detailed exposition of these calculations and a comparison with experiment will be given in a separate paper.

An indication that the effect in question does exist can be found, for example, in the experimental paper[7] reporting an investigation of inelastic scattering of 35-MeV protons by ^{208}Pb nucleus. The differential cross sections for the excitation of a large number of levels were measured, including nine levels with anomalous parity (three sets of levels 2^-, 4^-, and 6^-). In all nine cases, a noticable maximum was observed in the cross sections at the scattering angle $\sim 60°$ corresponding to the momentum transfer $k \sim p_F$ [2)] (the cross section here is 1.5–2 times larger than at the nearest minimum). Analogous effects should be observed not only in the inelastic scattering of the nucleons, but also in reactions with He^3, t, and other more complicated nuclei.

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[1)]We note that this coherent contribution is lost when $g_s(\mathbf{k})$ is calculated within the framework of the standard random-phase approximation with a truncated single-particle basis.

[2)]We note that at the same time there is no regular maximum in this angle region for states of normal parity.

[1]A. B. Migdal, Zh. Eksp. Teor. Fiz. **61**, 2209 (1971) [Sov. Phys. JETP **34**, 1184 (1972)]; A. B. Migdal, O. A. Markin, and I. N. Mishustin, Zh. Eksp. Teor. Fiz. **70**, 1592 (1976) [Sov. Phys. JETP **43**, 830 (1976)].
[2]E. E. Sapershteĭn, S. V. Tolokonnikov, and S. A. Fayans, Abstracts, 27th Conf. on Nuclear Spectroscopy, Tashkent, 1977, p. 412; M. A. Troitskiĭ, M. V. Koldaev, and N. I. Chekunaev, Pis'ma Zh. Eksp. Teor. Fiz. **25**, 136 (1977) [JETP Lett. **25**, 123 (1977)].
[3]A. B. Migdal, G. A. Sorokin, O. A. Markin, and I. N. Mishustin, Phys. Lett. **65B**, 423 (1976).
[4]A. B. Migdal, Teoriya konechnykh fermi-sistem i svoĭstva atomnykh yader (Theory of Finite Fermi Systems and Properties of Atomic Nuclei), Nauka, 1965 [Wiley, 1967].
[5]É. E. Sapershteĭn and M. A. Troitskiĭ, Yad. Fiz. **22**, 257 (1976) [Sov. J. Nucl. Phys. **22**, 132 (1975)].

[6]E. E. Saperstein, S. A. Fayans, and S. V. Tolokonnikov, J. Phys. G. **3**, 51 (1977); E. E. Sapershteĭn, S. V. Tolokonnikov, and S. A. Fayans, Yad. Fiz. **25**, 959 (1977) [Sov. J. Nucl. Phys. **25**, in press (1977)].
W. T. Wagner, G. M. Crawley, G. R. Hammerstein, and N. McManus, Phys. Rev. **C12**, 757 (1975).