Anomalies in static magnetoresistance due to magnetic surface levels

D. A. Romanov and L. D. Shvartsman
Institute of Semiconductor Physics, Siberian Branch, Academy of Sciences of the USSR

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The anomalous oscillations of the static magnetoresistance observed experimentally by Dolgopolov and Murzin {Pis'ma Zh. Eksp. Teor. Fiz. **37**, 468 (198) [JETP Lett. **37**, 554 (1983)]} and Skok *et al.* {Pis'ma Zh. Eksp. Teor. Fiz. **37**, 584 (1983) [JETP Lett. **37**, 696 (1983)]} are explained on the basis of a qualitative change in the spectrum of carriers in magnetic surface levels. This change is caused by an enrichment near the surface.

There have been several recent reports of the observation of unusual oscillations in the static conductivity of semimetal samples and degenerate semiconductor samples in magnetic fields.[1,2] Dolgopolov and Murzin,[1] for example, observed the appearance of an additional period in the Shubnikov–de Haas oscillations in bismuth samples. The additional oscillations are nearly periodic in the reciprocal magnetic field. The effect is sensitive to the surface properties of the sample and is modulated by a transverse electric field.

We know that oscillations of the static magnetoresistance are always caused by singularities in the state density. *Ordinary* magnetic surface levels do not make a singular contribution to the state density different from the contribution of bulk Landau levels. As we will show below, an enrichment potential near the surface gives rise to additional critical (extremal) points in the carrier dispersion law, and these features may be manifested as substantially more complex Shubnikov–de Haas oscillations.

We consider a degenerate electron gas with a quadratic dispersion law. Motion along the Y axis, which is directed into the sample, is prevented by the infinite wall at the surface of the sample and the resultant potential. This potential is formed by the enrichment layer, which we will call $-u(Y)$ $[u(Y_1)=0]$, and by the part of the parabola of the effective potential energy of an electron in the magnetic field, $m\omega_c^2(Y-Y_0)^2/2$, where $Y_0=-cp_x/e\mathscr{H}$. [For simplicity, we are assuming that m is isotropic. For a particular form of m_{ik} (for Bi, for example), the results are modified by the introduction of corresponding angular coefficients.] In the gauge adopted here, $\mathbf{A}(-\mathscr{H}Y,0,0)$ the magnetic field $\mathscr{H}\|\mathbf{z}$ is parallel to the xz plane of the boundary of the sample. It is convenient to introduce $y=Y\sqrt{m\omega_c^2/2}$, so that y^2 has the dimensionality of an energy. Restricting the analysis to a semiclassical treatment, which is adequate in the cases of interest here, we write the quantization condition as

$$\int_0^{\sqrt{\cdots}=0}\sqrt{\epsilon+u(y)-(y-y_0)^2}\,dy=\frac{\pi\hbar\omega_c}{2}N\,. \tag{1}$$

Here ϵ is the energy of the transverse motion, and $N=n+3/4$.

The energy position of the critical points, with $\partial\epsilon_n/\partial p_x = 0$, is determined by Eq. (1) and the condition $\partial\epsilon/\partial y_0 = 0$, which can be written as follows, where we are using (1):

$$\int_0^{\sqrt{\cdots}=0} d\left(\sqrt{\epsilon + u(y) - (y-y_0)^2}\right) - \frac{1}{2}\int_0^{\sqrt{\cdots}=0} dy \frac{du}{dy} \frac{1}{\sqrt{\epsilon + u(y) - (y-y_0)^2}} = 0. \tag{2}$$

For a slight enrichment near the surface we would have

$$\beta = \frac{y_1}{y_0} \ll 1; \quad \alpha = \left|\overline{\frac{du}{dy}}\right| / 2y_0 \ll 1 \tag{3}$$

(the superior bar means an average over the interval $[0,y_1]$), the solution of (2) is determined by the integral on the right in first approximation:

$$y_0^2 = \epsilon + u(0). \tag{4}$$

The correction from the second integral is on the order of $y_1\alpha^2$ and is negligibly small, by virtue of (3). Substituting y_0 from (4) into (1), singling out the integral over the enrichment region, expanding its integrand, and making use of the small quantity α, we find an algebraic equation that determines $\epsilon(n)$ at the critical points (within terms $\sim\sqrt{\alpha}$):

$$\epsilon\frac{\pi}{2} + \frac{C}{2\sqrt{2}}\epsilon^{-1/4} u(0)\sqrt{y_1} = \frac{\pi}{2}\hbar\omega_c N, \tag{5}$$

where the numerical factor

$$C = \frac{1}{u(0)\sqrt{y_1}} \int_0^{y_1} \frac{u(y)}{\sqrt{y}} dy \tag{6}$$

characterizes the shape of the surface potential.

The period T of the additional oscillations in the static magnetoresistance along the scale of the reciprocal magnetic field can be determined directly from (5). For this purpose, we transform to the ordinary variables Y and set $\epsilon = \epsilon_F$:

$$T = \frac{e\hbar}{mc}\left(\frac{\partial N}{\partial(1/\hbar\omega_c)}\right)^{-1}$$

$$= \frac{e\hbar}{mc}\left[\epsilon_F + \frac{C}{2\pi\sqrt{2}} u(0)\left(\frac{mY_1^2\omega_c^2}{2\epsilon_F}\right)^{1/4}\right]^{-1}. \tag{7}$$

The conductivity oscillations which result from the "extra" critical points of the magnetic surface levels are thus periodic in $\mathscr{H}^{-1}$ in a first approximation. The relative correction to the period depends in a fundamental way on the magnetic field and is given by (in a convenient notation)

$$\frac{\Delta T}{T} = -\frac{C}{2\pi\sqrt{2}}\;\frac{u(0)}{\epsilon_F}\;\frac{\sqrt{\lambda_F y_1}}{l_{\mathscr{H}}}, \tag{8}$$

where $l_{\mathscr{H}} = \sqrt{c\hbar/e\mathscr{H}}$ is the magnetic length, and λ_F is the electron wavelength at the Fermi level. For the particular properties of bismuth this correction would be 2–3%, in good agreement with experimental results.

The case described here is apparently close to the actual situation in the experiments of Ref. 1. For the other case of a "deep" potential ($\alpha \gg 1, \beta \ll 1$), which is encountered in semiconductors, similar calculations lead to the following correction to the period in $\mathscr{H}^{-1}$:

$$\frac{\Delta T}{T} = b\,\frac{m\,Y_1^2}{\hbar^2\sqrt{\epsilon_F u(0)}}\,(\hbar\omega_c)^2, \tag{9}$$

where the numerical factor is

$$b = \frac{\sqrt{u(0)}}{Y_1^2}\int_0^{Y_1}\frac{(Y - Y_1)}{\sqrt{u(Y)}}\,dY. \tag{10}$$

Finally, if the enrichment potential near the surface is both "deep and wide" ($\beta \sim 1$), the semiclassical period of the transit of a particle over the potential is short, and the energy spacings between the additional critical points near ϵ_F may be substantially greater than $\hbar\omega_c$ (Ref. 3). As a result, oscillations in the static magnetoresistance in magnetic fields will be so weak that the bulk Shubnikov–de Haas oscillations will still be masked. A similar effect was apparently observed by Skok *et al.*[2]

We might note in conclusion that the existence of additional critical points in the dispersion law of magnetic surface levels may be manifested in effects other than those described above if these effects are sensitive to the structural features in the state density. Possible examples are the optical magnetoabsorption spectrum and the photovoltaic effect.

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[1]V. T. Dolgopolov and S. S. Murzin, Pis'ma Zh. Eksp. Teor. Fiz. **37**, 468 (1983) [JETP Lett. **37**, 554 (1983)].
[2]S. S. Murzin and V. T. Dolgopolov, Pis'ma Zh. Eksp. Teor. Fiz. **37**, 584 (1983) [JETP Lett. **37**, 696 (1983)].
[3]D. A. Romanov and L. D. Shvartsman, Tezisy XI Soveshaniya po teorii poluprovodnikov (Abstracts, Eleventh Conference on the Theory of Semiconductors), Uzhgorod, 1983.

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