Photopositronium in the magnetosphere of a pulsar

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A mixed state of a photon and positronium forms in a strong magnetic field. The corresponding dispersion curves are described. This effect makes it possible to raise the estimated luminosity of pulsars to the observed values if the magnetic fields at the surfaces of the pulsars are stronger than 4×10^{12} G.

According to the present understanding,[1–3] the following processes occur in the magnetosphere of a pulsar and ultimately determine the emission of the pulsar. The electric field **E** created by the rotation of the pulsar's magnetic field **B** accelerates charged particles near the surface of the pulsar. These particles move along the curved magnetic lines of force and radiate "bending" γ rays tangent to the lines of force of the field **B** (Refs. 1,2, and 4). A resulting γ ray produces two particles, an electron and a positron, each in a Landau level but moving freely along **B**. Beginning at a certain distance from the surface, these particle pairs screen the field **E**.

The last part of this picture changes substantially if we take into account the appearance of a mixed state of a photon with a Coulomb-bound pair (a positronium

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atom). We call this state "photopositronium" by analogy with the term "photoexciton." It forms because single-photon production of positronium is kinematically allowed in a magnetic field, as was discussed in Refs. 5 and 6 and as has recently been reemphasized by Leinson and Oraevskiĭ[7] (who thereby stimulated the present letter). As a bending γ ray propagates, it evolves along the dispersion curve of the photopositronium, converting adiabatically into positronium. The energy of the photon is channeled along the direction of the curved magnetic field. This is an amplification of our earlier results,[8–11] where an analogous but quantitatively weaker capture was found in a study of a mixed state of a photon with an e^+e^- pair at the boundary of the continuous spectrum. The most important distinction in the present case is the appearance of a binding energy which is logarithmically large in proportion to B (a collapse toward the center in the Coulomb-bound system in the limit $B \to \infty$; Ref. 12). This circumstance makes the positronium atoms which are produced stable with respect to the ionizing effect of the field **E**. We will discuss here the relevance of this fact to the physics of pulsars.

For our purposes it is sufficient to consider the case $B \gg 2\times 10^9$ G, in which the wave function of the bound pair can be assumed to depend on the degrees of freedom across **B** in the same way as for a free pair. To find compatible dispersion curves for the photon and the positronium, we need to describe the latter by a complete set of quantum numbers of such a nature that it includes quantities which are related to the photon momentum $\hbar\mathbf{k}$ by the conservation laws that follow from the translational invariance of the polarization operator in a static field $B_z = B$, $B_x = B_y = 0$. Such a set of quantum numbers arises in a natural way if we use the gauge $A_x = -By, A_y = A_z = A_0 = 0$ for the potentials of the external field. In this gauge, the translational invariance along the x axis is explicit. Following Leinson and Oraevskiĭ,[7] who were the first to apply the Bethe-Salpeter equation to the problem of the spectrum of positronium in a magnetic field, we find a one-dimensional Schrödinger equation in the difference $z^- - z^+$ between the coordinates of the electron and the positron, which are in Landau levels n and n'. For the potential we find an extremely natural expression in this case:

$$V(z^- - z^+) = -\alpha[(z^- - z^+)^2 + (y_0^- - y_0^+)^2]^{-1/2}, \quad \alpha = 1/137, \tag{1}$$

where $(y_0^- - y_0^+) = (p_x^- + p_x^+)c/(eB)$ is the distance between the y coordinates of the centers of the orbits of the electron and the positron, and $(p_x^\pm/\hbar)$ are their wave vectors along the x axis. The discrete spectrum of values of the total energy of the positronium, $\epsilon_{nn'}(n_c, p_x^+ + p_x^-)$, is labeled for each n,n' by the quantum $n_c = 0, 1,\ldots,$ and is shown by the nearly horizontal thin curves in Fig. 1. Plotted along the abscissa here is the square of the component of the photon's momentum across the magnetic field, $\hbar^2 k_\perp^2 = \hbar^2 k_x^2$, which is related to the quantum numbers of the positronium produced by the photon by the conservation law $\hbar k_\perp = p_x^+ + p_x^-$ (we are choosing $k_y = 0$). Plotted along the ordinate is the square of the photon energy, $(\hbar\omega)^2 \equiv c^2\hbar^2 k_0^2$, minus the square of its momentum $(\hbar k_\parallel)$ along the field, which is equal to the momentum of the center of mass of the positronium in the same direction. The quantity $c^2\hbar^2(k_0^2 - k_\parallel^2)$ is equal to $\epsilon_{nn'}^2(n_c, p_x^+ + p_x^-)$ by virtue of energy conservation. Using the results of Ref. 12, we find the following expression for the lower spectral curve of

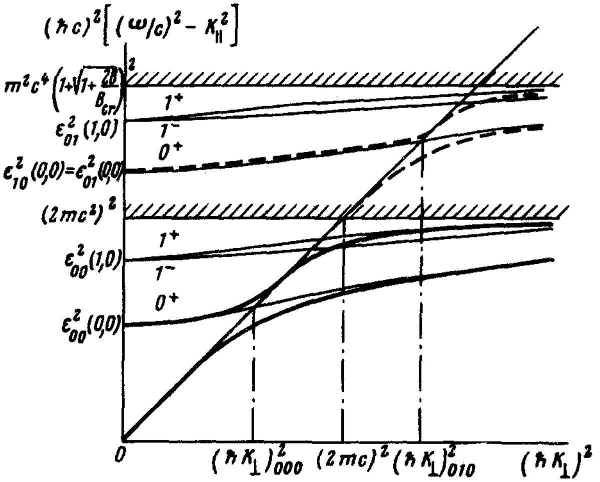

FIG. 1. Dispersion curves of a photon of positronium, and of their mixed state. The positronium curves are labeled with $n_c^\pm$ where $n_c = 0,1,\ldots$, and the $\pm$ signs specify the parity of the state under reflection of the z axis.

positronium with $n_c = 0$ for each n,n' series (this expression is valid as long as the logarithm is large):

$$c\hbar\left[\left(\frac{\omega}{c}\right)^2 - k_\parallel^2\right]^{1/2} = (m_n + m_{n'})c^2 - 2\alpha^2 M_{nn'} c^2 \left(\ln \frac{\beta m}{2\alpha M_{nn'}\sqrt{\beta+\eta^2}} \right)^2$$
$$= \epsilon_{nn'}(0, k_\perp \hbar), \tag{2}$$

where $\beta = B/B_{cr}, B_{cr} = (m^2c^3/e\hbar) = 4.4\times 10^{13}$ G, $\eta = \hbar k_\perp/mc$, and $M_{nn'} = m_n m_{n'}/(m_n + m_{n'})$ is the reduced mass of the particles in Landau levels n,n' with masses $m_j = m\sqrt{1+2j\beta}, j = n,n'$. The dashed horizontal lines show the boundaries of the continuum in the cases $n = n' = 0$ (the lower line) and $n = n' = 1$ (the upper line). The sloping straight line here shows the dispersion curve of the photon if we ignore the polarization of vacuum, $k_0^2 - k_\parallel^2 = k_\perp^2$. The intersections of this line with the spectral curves of the positronium and with the boundaries of the continuum are quasiintersections (for states which are of even parity under the reflection $z^+ \longleftrightarrow z^-$). Near these quasiintersections we cannot use perturbation theory; the dispersion curves of the photon and the positronium interact strongly, repelling each other and reconnecting. Generalizing the procedure of Ref. 9, we find a representation of the eigenvalues of the polarization operator $\kappa_i, i = 1,2,3$ as the sum of the pole contributions of the positronium states, and we solve the dispersion relation $k_0^2 - k_\parallel^2 - k_\perp^2 = \kappa_i(k_0^2 - k_\parallel^2, k_\perp^2)$ near the quasiintersections. With $n = n' = 0$, only the photon of mode $i = 2$, whose electric vector $\mathbf{e}$ lies in the plane formed by the vectors $\mathbf{k}$ and $\mathbf{B}$, combines with the positronium, forming photopositronium, for which the dispersion curves (the heavy lines in Fig. 1) are described by

$$(c\hbar)^2\,(k_0^2 - k_\parallel^2)_\pm = 2m^2c^4\left[E^2 + \frac{\eta^2}{4} \pm \sqrt{\left(E^2 - \frac{\eta^2}{4}\right)^2 + 4\alpha\beta E\left(\frac{1-E}{2}\right)^{1/2}\exp\left(-\frac{\eta^2}{2\beta}\right)}\right], \tag{3}$$

where $E = \epsilon_{00}(0,k_\perp \hbar)/2mc^2$. The dashed lines in Fig. 1 show the dispersion curves for the other mode, with $\mathbf{e}\perp\mathbf{k}$ and $\mathbf{e}\perp\mathbf{B}$.

In the range of applicability of geometric optics, a γ ray emitted along the tangent to a curved magnetic line of force experiences changes in its components $k_\parallel$ and $k_\perp$ during its propagation. It moves along the lower branch of the dispersion curve in Fig. 1 upward and to the right, gradually undergoing a genuine conversion into positronium at $k_\perp^2 \gg (2mc/\hbar)^2$. The center of the wave packet bends in the direction in which the lines of force are curved, since the component of the group velocity across the field, $c(\partial k_0/\partial k_\perp)$, tends toward zero with increasing $k_\perp$. The analytic expressions determining the motion of the center of the packet are the same as those in Refs. 10 and 11, except that the function $k_\parallel(k_\perp)$ is taken from (3). It can be shown that at $B \gtrsim 0.1B_{cr}$ the rotation occurs quite smoothly, so that the geometric-optics approximation holds. The instability of positronium with $n = n' = 0$ due to processes distinct from single-photon annihilation must be taken into account in this context as a broadening of its spectral line. It can be shown that again under the condition $B \gtrsim 0.1B_{cr}$ the broadening due to the main process—the decay into two photons (cf. Ref. 13), which is allowed to the left of the intersection of the dispersion curve with the line $k_0^2 - k_\parallel^2 = k_\perp^2$—does not lead to a filling of the gap between the + and − branches of curves (2). In weaker fields there may be a leakage from one branch to the other , and unbound e^+e^- pairs may ultimately be produced. These pairs will screen the electric field of the pulsar. This happens at distances from the pulsar at which the magnetic field has become weak ($B \lesssim 0.1B_{cr}$). This circumstance determines the height (h) of the polar gap in our model, i.e., the region near the pulsar in which the electric field accelerates charged particles, pumping energy into the magnetosphere. For the pulsar PSR 0833-45, near the remnants of the Vela supernova, with a surface magnetic field $B_s = 7\times 10^{12}$ G, the height of the polar gap turns out to be $h \simeq 2\times 10^5$ cm or more than an order of magnitude higher than the value found by Arons[3] on the conventional grounds that all the pairs produced are not coupled with each other at arbitrarily strong fields B. The luminosity expected theoretically for the pulsar is proportional to h, so that the effect discussed here makes it possible to raise the estimated luminosity of PSR 0833-45 from the 2×10^{33} erg/s in Ref. 3 to the observed value[14] of 4×10^{34} erg/s. We might note that the electric field in the gap, $\sim 10^5$ esu, is considerably lower than the value $E_\parallel^{\text{ion}} \simeq 4\times 10^7$ esu which is required for the ionization of positronium.

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