Hydrodynamic theory of echo on surface waves

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An echo which is generated by a surface wave and which is hydrodynamic in nature can occur in a bounded plasma with an inhomogeneous transition layer. Continuous plasma waves which are launched in the transition layer retain the information about the external perturbations.

Romanov[1] and Stepanov[2] have shown that a surface wave in a plasma with a diffuse boundary undergoes a collisionless damping due to a transfer of its energy to longitudinal plasma waves near the plasma resonance where the frequency of the surface wave is equal to the local plasma frequency. The analysis of Dolgopolov and Omel'chenko[3] has shown that after the surface wave is damped, the oscillation of the electric-field component parallel to the density gradient remains undamped in the transition layer of a cold collisionless plasma. In a plasma with a diffuse boundary the resonant damping of a surface wave thus has no connection with the irreversible energy dissipation of the wave motion. In this letter we show that plasma-echo effects can occur in a system of this sort. The undamped oscillations of the electric field near the plasma resonance serve as the Van-Campen waves in this case.[4]

We assume that the plasma occupies a region $x > 0$. We also assume that the equilibrium plasma density $n_0(x)$ increases monotonically in the transition region $0 < x < a$ and that it is constant at $x > a$. Restricting the discussion to the potential oscillations in the cold plasma with stationary ions, we begin with the system of hydrodynamics equations

$$\Delta\phi = 4\pi e n, \quad \frac{\partial n}{\partial t} + \mathrm{div}(n\mathbf{v}) = 0,$$
$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v}\vec{\nabla})\mathbf{v} = \frac{e}{m}\nabla\phi. \tag{1}$$

Carrying out a Laplace transformation with respect to time and a Fourier transformation with respect to the y coordinate directed along the plasma boundary, in first order perturbation theory we find from (1) the following equation for the potential:

$$\frac{\partial}{\partial x}\left(\epsilon \frac{\partial}{\partial x}\phi^{(1)}_{kp}\right) - k^2\epsilon\,\phi^{(1)}_{kp} = \frac{4\pi e}{p} G_{kp}, \tag{2}$$

where $\epsilon(x,p) = 1 + \omega^2_{Le}(x)/p^2$ is the dielectric constant of a cold plasma, and

$$G_{kp}(x) = \left(n^{(1)}_k + \frac{1}{p}\frac{\partial}{\partial t} n^{(1)}_k\right)\Big|_{t=0}$$

is the initial density perturbation which is regarded below as an external perturbation.

Joining the solutions of Eq. (2) obtained in three regions ($x<0$, $0<x<a$, and $x>a$) by making use of the conditions under which ϕ and $\partial\phi/\partial x$ are continuous at the points $x=0$ and $x=a$, we find the following expressions for the potential and for the component of the electron velocity which is parallel to the density gradient ($ka \ll 1$):

$$\phi^{(1)}_{kp}(x) = -\frac{4\pi e}{pkD(p,k)}\left(1 + k\int_0^x \frac{dx'}{\epsilon(x',p)}\right)\int_0^a G_{kp}(x)\,dx, \tag{3}$$

$$v^{(1)}_{xkp}(x) = -\frac{4\pi e^2}{mp^2\,\epsilon(x,p)D(p,k)}\int_0^a G_{kp}(x)\,dx. \tag{4}$$

Here

$$D(p,k) = 1 + k\int_0^a \epsilon(x,p)\,dx + \epsilon(a,p)\left(1 + k\int_0^a \frac{dx}{\epsilon(x,p)}\right) \tag{5}$$

is the dispersion function of the surface waves whose poles

$$p = \pm i\omega_0 - p_k,$$
$$\omega_0 = \omega_{Le}(a)/\sqrt{2}, \quad p_k = \frac{1}{4}\pi k h\omega_0, \quad h \sim a \tag{6}$$

describe the waves that decay exponentially over time.[2,3] From (3) we see that oscillations of the potential caused by external perturbations damp over time. In contrast, $v^{(1)}_x$ in the transition layer undergoes continuous oscillations which are described by the poles $\epsilon(x,p) = 0$.

Solving in an analogous manner the system of equations (1) in second-order perturbation theory, we find the nonlinear potential $\phi^{(2)}(x)$ and the nonlinear surface charge

$$\sigma_{kp}^{(2)} = \int_0^a n_{kp}^{(2)}(x)\,dx$$

$$= -\frac{k}{2p^2}\frac{2\epsilon(a,p) - D(p,k)}{D(p,k)} \int_0^a dx \frac{\partial}{\partial x}\left(\frac{n_0(x')}{\epsilon(p,x)}\right)\int \frac{dk'}{2\pi}\int\frac{dp'}{2\pi i} \,\tilde{v}^{(1)}_{xk-k',p-p'}\, v^{(1)}_{xk'p'}\,. \qquad (7)$$

The initial density perturbations can be expressed as signals with a spatial dependence of the form $\exp(\pm ik_1 y)$ and $\exp(ik_2 y)$ which arrive in the plasma at the times $t=0$ and $t=\tau$, respectively:

$$G_{kp}(x) = 2\pi n_1(x)\delta(k \pm k_1) + 2\pi n_2(x)\, e^{-p\tau}\,\delta(k-k_2). \qquad (8)$$

Substituting relations (4) and (8) into (7), we find the following expression for the surface-charge density of an echo signal:

$$\sigma_{\pm}^{(2)}(y,t) \cong \frac{\pi e^2}{6m} N_1 N_2 k_{\pm}(t-\tau)\, e^{ik_{\pm}y}\int_0^{\omega_L e(a)} \frac{d\omega}{\omega^2}\left[i\,\frac{2\epsilon(a,i\omega) - D(i\omega,k_{\pm})}{D(i\omega,k_{\pm})}\right.$$

$$\left.\times\frac{\exp[i\omega(t-2\tau)]}{D(2i\omega,k_2)D(-i\omega,\pm k_1)} + \text{c.c.}\right], \qquad (9)$$

where $k_{\pm} = k_2 \pm k_1$, $N_i = \int_0^a n_i(x)dx$.

Charge (9) perturbs the field of the echo surface wave whose potential at the boundary is

$$\phi_{\pm}^{(2)}(0) \approx \phi_{\pm}^{(2)}(a) \approx 4\pi\sigma_{\pm}^{(2)}/k_{\pm}^2 a\,. \qquad (10)$$

Integrating in expression (9), we finally find ($p_i = p_{k_i}$, $\omega_0|t-2\tau| \gtrsim 1$)

$$\sigma_{\pm}^{(2)}(y,t) \cong \left(\frac{\pi e}{6}\right)^2 \frac{2N_1N_2\,k_{\pm}(t-\tau)}{m\,(p_1+p_{\pm})}\, e^{ik_{\pm}y}\sin[\omega_0(t-2\tau)]$$

$$\times\begin{cases}\exp[p_1(t-2\tau)], & t<2\tau\\ \exp[-p_{\pm}(t-2\tau)], & t>2\tau\end{cases}. \qquad (11)$$

From (11) we see that the echo signal peaks at the time $t=2\tau$ and decays exponentially on both sides of the peak. The shape of the signal is nonsymmetric with respect

to time. The echo appears either with a difference wave number k_- or a sum wa
number k_+.

In summary, an echo which is generated by a surface wave in a bounded plasn
with an inhomogeneous transition layer is hydrodynamic in nature and is intrinsical
analogous to an oscillation echo occurring in systems with a continuous spectrum.

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