Suppression of gross plasma instabilities in axisymmetric tandem mirrors

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The use of a short (nonparaxial) mirror system as the end cell in a tandem mirror makes it possible to stabilize the most dangerous MHD perturbations even in an axisymmetric configuration.

Axisymmetric tandem mirrors ("ambipolar traps") have several important advantages over axially asymmetric systems.[1,2] Attempts to develop them, however, run into the serious difficulty of stabilizing the flute waves which are excited in these systems by the unfavorable (on the average) curvature of the magnetic lines of force. Only in recent years have several ways been suggested[3–9] to stabilize these waves in axisymmetric tandem mirrors. In the present letter we suggest yet another way to solve this problem.

This new stabilization method is oriented toward the conventional layout of the tandem mirror, with a long central mirror system and two comparatively short end mirror systems. The flute waves with azimuthal number $m \geqslant 2$ are assumed to be stabilized by the finite Larmor radius in the central cell. The only remaining problem is then to stabilize the $m = 1$ mode, which corresponds to a displacement of the plasma as a whole and which is thus insensitive to the finite-Larmor-radius effects (see Ref. 10, for example). We propose stabilizing this mode by making use of the nonparaxial nature of the magnetic field in the end cells (for this purpose, the transverse dimension of the plasma in these cells must be comparable to their length). An important point is that the stabilization occurs in the "natural" geometry of open confinement systems and does not require switching to a non-singly-connected plasma.

To get an idea of the possibilities of this stabilization mechanism, we consider a model in which the plasma in an end cell is treated as an isotropic gas with an adiabatic index $\gamma = 5/3$, while the mirror ratio in the end cell is very large (so that the properties of the plasma in the end and central cells can be assumed independent of each other). The stability conditions derived from this model are sufficient conditions.

Let us find the perturbation of the potential energy in an end cell in the case of a flute displacement. In accordance with the discussion above, we assume that the structure of the eigenfunction which determines the displacement of the plasma is specified by the requirement that finite-Larmor-radius effects in the central cell must be minimized. This requirement corresponds to a displacement

$$\vec{\xi}_\perp = \frac{\text{const}}{B^2}[\mathbf{B}, \nabla\psi]; \quad \psi = \text{const}\sqrt{\Phi}\cos\varphi, \qquad (1)$$

where ψ is the electrostatic potential, Φ is the magnetic flux inside the given magnetic

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surface, and φ is the azimuthal angle. Substituting displacement (10) into the standard expression for the potential energy,[10] we find

$$\delta W = A \int \Phi \left[\frac{dp}{d\Phi} \frac{dU}{d\Phi} + \gamma \frac{p}{U} \left(\frac{dU}{d\Phi} \right)^2 \right] d\Phi , \qquad (2)$$

where $U = \int dl/B$, and $A > 0$ is a normalization constant of no importance to the discussion below.

If the transverse dimension of the plasma is small in comparison with the longitudinal scale length for changes in the magnetic field, we can ignore the second term in square brackets; we find that the plasma is unstable in a universal way—regardless of the pressure profile $p(\Phi)$. The last term, which plays a stabilizing role, becomes important at distances from the axis comparable to the length of the central cell.

Any radial pressure profile can be "composed" from δ-functions of the type

$$p = p_0 \, \delta (\Phi - \Phi_0) . \qquad (3)$$

Since expression (2) is linear in p, we can prove that the existence of stable pressure profiles is possible by showing that the condition $\delta W > 0$ holds for at least one position of a pressure peak of the type in (3). Substituting (3) into (2), and integrating by parts in the first term, we find

$$\delta W = A p_0 \left[\gamma \frac{\Phi}{U} \left(\frac{dU}{d\Phi} \right)^2 - \frac{d}{d\Phi} \left(\Phi \frac{dU}{d\Phi} \right) \right] \equiv A p_0 F(\Phi) .$$

The function F is negative at small values of Φ by virtue of the condition $dU/d\Phi > 0$. At large values of Φ the confinement region is usually bounded by a separatrix which passes through the null point(s) of the magnetic field (shown as an example in Fig. 1 are the lines of force of the magnetic field resulting from a superposition of the field of two "point" mirror coils at points A and B and a uniform longitudinal field with a strength 6×10^{-2} of that of the field of the point coils at point 0). It can be shown that the function F is also negative (tends toward $-\infty$) at the outer boundary

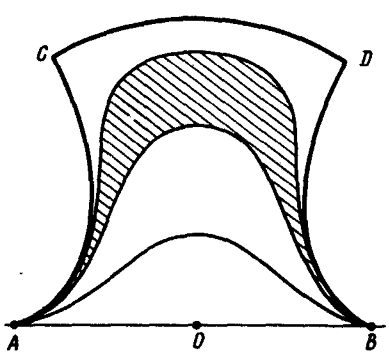

FIG. 1. Magnetic lines of force. The heavy line is the separatrix. The magnetic field vanishes at points C and D. The hatched region is the "stability ring."

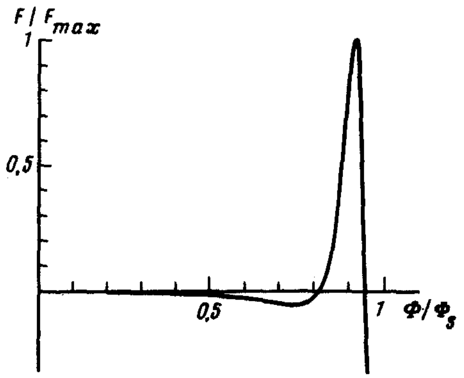

FIG. 2. The function $F(\Phi)$, normalized to a unit maximum value F_{max}.

of the confinement region as $\Phi \to \Phi_s$, where Φ_s is the magnetic flux inside the separatrix. Accordingly, stable states can exist only at intermediate values of Φ, in an annular region. Remarkably, if such a region ("stability ring") does exist, then *any* pressure distribution will be stable inside it.

Figure 2 shows the function $F(\Phi)$ corresponding to the magnetic field in Fig. 1. We see that a stability ring does in fact exist in this case.

By producing a plasma in the stability ring, we provide a margin of MHD stability which makes it possible to also place a certain amount of plasma in the axial region of the end mirror cells. For the magnetic configuration in Fig. 1, for example, a numerical calculation shows that "step" pressure profiles ($p = \text{const}$ at $\Phi < \Phi_1$ and $p = 0$ at $\Phi > \Phi_1$, $0.89 < \Phi_1/\Phi_s < 0.96$ are stable. The presence of plasma in the axial region may be important for an ambipolar plugging of the central cell.

In summary, this study shows that the most dangerous ($m = 1$) mode of flute waves in a tandem mirror can be stabilized by means of simple axisymmetric end mirror systems. Pressure profiles without an axial dip are permissible.

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