

Density distribution in solid He⁴ grown in a closed capillary

I. A. Gachechiladze and D. V. Pavlov

(Submitted 10 April 1985)

Pis'ma Zh. Eksp. Teor. Fiz. **42**, No. 2, 45–47 (25 July 1985)

The growth of a He⁴ crystal in a closed volume (by the blocked-growth method) is studied. The density distribution of the solid phase along the axis of a long cylindrical container originally filled with a liquid under pressure is derived.

The "blocked-capillary method" is widely used to produce solid helium. Although it is assumed here that the resulting samples have a homogeneous density equal to that of the original liquid,^{1–4} the validity of this assumption is not obvious when crystals are grown in narrow capillaries.

In this letter we study the growth of a He⁴ crystal in a closed cylindrical container filled beforehand with liquid He⁴ at a pressure $P_0 > 25$ atm.

We lower the temperature of one end of the container to the solidification temperature for the given value of P_0 . Crystallization of the helium begins at the cooled end surface. Under the condition $L \gg R$, where L and R are the length and inside radius of the container, we can assume that the interface between the solid and liquid phases is plane and perpendicular to the axis of the container. We direct the z axis along this axis; the positive z direction is the direction in which the boundary moves, and the origin is put at the plane of the cooled end.

Since the density (ρ_s) of solid He⁴ is higher than that (ρ_l) of the liquid at the same pressure, the growth of the solid phase toward the liquid must be accompanied by an acquisition of mass from the liquid phase, and the pressure of the liquid must decrease continuously as the crystal grows. The density of the solid helium depends in turn on the crystallization pressure, so that a crystal with a density $\rho_s(Z)$, which is inhomogeneous along its length, grows in the closed volume.

Let us find the density profile $\rho_s(Z)$ under the assumption that no redistribution of the density in the solid phase occurs. We will not make the further assumption that the difference $\Delta\rho = \rho_s - \rho_l$ varies along the melting curve, since this change does not exceed 4% between 30 and 120 atm (Ref. 5).

We assume that a layer of solid phase of infinitesimal thickness dz has grown on a crystal of length $Z_1 < L$. Mass conservation leads to the differential equation $d\rho_l(z_1)/dz_1 = -\Delta\rho/(L - z_1)$. Denoting by $\rho_l(0)$ the initial density of the liquid, we find

$$\rho_l(z_1) = \rho_l(0) + \Delta\rho \ln(1 - z_1/L). \quad (1)$$

The initial density of the crystal is $\rho_s(0) = \rho_l(0) + \Delta\rho$, so that under the assumptions above the density distribution in the solid phase is

$$\rho_s(z) = \rho_s(0) + \Delta\rho \ln(1 - z/L), \quad 0 \leq z \leq z_1. \quad (2)$$

Andronikashvili *et al.*² have measured the resonant frequency ω of bending vibrations of a cylindrical container ($L = 30$ mm, $R = 0.15$ mm), in which He⁴ is solidified

by the blocked-capillary method. The growth of the crystal at the fixed and cooled end leads to a flow of mass of the liquid away from the vibrating free end; the effect is to raise the resonant vibration frequency (the points in Fig. 1).

Under the condition $\rho_l < \rho_s \ll \sigma/\pi R^2$ the relative shift of the square of the resonant frequency is given by

$$\Delta\omega^2(z_1)/\omega^2 = -(\pi R^2/\sigma) \int_0^{z_1} x^2(z) \rho(z) dz / \int_0^L x^2(z) dz, \quad (3)$$

where σ is the linear density of the unfilled container, the function $x(z)$ describes the shape of a uniform rod undergoing bending vibrations,⁶ and we have $\rho(z) = \rho_s(z)$ at $0 < z < z_1$ and $\rho(z) = \rho_l(z_1)$ at $z_1 < z < L$.

The position of the interface, $z_1(T)$, can be related in an unambiguous way to the temperature with the help of the known temperature dependence $\tilde{\rho}_s(T)$ along the melting curve⁵ and Eq. (2):

$$z_1(T)/L = 1 - \exp[(\tilde{\rho}_s(T) - \rho_s(0))/\Delta T]. \quad (4)$$

The results of a numerical calculation of the frequency shift from Eqs. (3) and (4) are shown by the solid line in Fig. 1. The dashed line shows the calculated behavior of the relative distance over which the crystal grows, $z_1(T)/L$. In the temperature interval $1.8 < T < 3.4$ K (3.4 K is the crystallization temperature at an initial pressure of 95 atm) we observe a satisfactory agreement between the calculated and measured frequency shifts. The sharp decrease in the frequency observed at $T = 1.78$ K results from the nucleation of the γ phase, at which point a three-phase system appears in the container. The further growth of the He^4 crystal thus cannot be analyzed by the model used here.

If we ignore the appearance of the γ phase, however, we find from Eq. (1) that at a certain z_1 the density of the liquid reaches a value corresponding to a pressure of 25 atm, and the crystal growth comes to a halt. Although the growth of the γ phase causes a significant redistribution of mass, this redistribution does not cause a com-

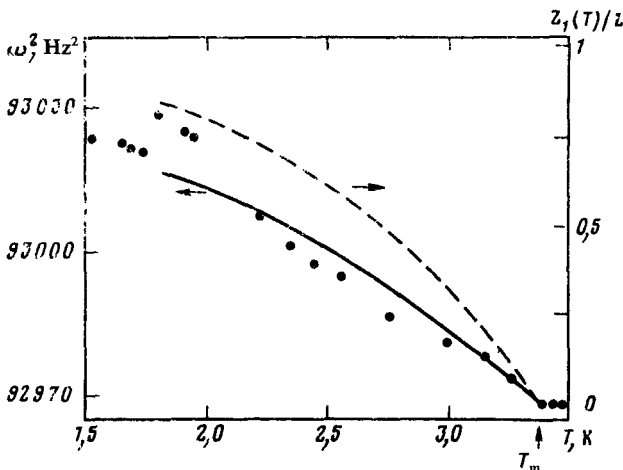


FIG. 1. Temperature dependence of the resonant frequency of a helium-filled capillary. $P_0 = 95$ atm. Points—Experimental data; solid line—calculations from Eqs. (3) and (4); dashed line—relative distance over which the crystal grows according to calculations from Eq. (4).

plete equalization of the density, as can be seen from the experimental data in Fig. 1. Accordingly, the helium may not fill the entire length of the container in crystallization by the blocked-capillary method.

If the density became completely uniform over the entire length of the container, the vibration frequency of the container would have to return to its original value (under the condition that the mass of helium inside the container is constant) or even drop below this original value (if additional mass is forced out of the filling line). Since we do not observe this effect, we conclude that the redistribution of density in the crystals grown in a blocked container with the specified dimensions is insignificant over a time on the order of several hours.

The shear stresses in the crystal corresponding to the calculated density gradient reach $(1-5) \times 10^5$ dyn/cm². This value is an order of magnitude higher than the values at which plastic flow of solid He⁴ has been observed in experiments with bulk samples.⁷

In summary, the model proposed here gives a qualitatively correct description of the growth of a He⁴ crystal in a closed volume with the specified values of L and R . We believe that questions related to the microstructure of the crystals of variable density, the reasons for the anomalous strength of these crystals, and the mechanism for the growth of the γ phase require a special study.

¹V. N. Grigor'ev, B. N. Esel'son, and V. A. Mikheev, *Zh. Eksp. Teor. Fiz.* **64**, 608 (1973) [*Sov. Phys. JETP* **37**, 309 (1973)].

²É. L. Andronikashvili, I. A. Gachechiladze, and V. A. Melik-Shakhnazarov, *Fiz. Nizk. Temp.* **1**, 635 (1975) [*Sov. J. Low Temp. Phys.* **1**, 305 (1975)].

³M. A. Paalanen, D. J. Bishop, and H. W. Dail, *Phys. Rev. Lett.* **46**, 664 (1981).

⁴D. J. Bishop, M. A. Paalanen, and J. D. Reppy, *Phys. Rev. B* **24**, 2844 (1981).

⁵E. R. Grilly and R. L. Mills, *Ann. Phys.* **8**, 1 (1959).

⁶L. D. Landau and E. M. Lifshitz, *Teoriya uprugosti*, Nauka, Moscow, 1965, p. 149 (*Theory of Elasticity*, Pergamon, New York, 1970).

⁷V. L. Tsymbalenko, *Zh. Eksp. Teor. Fiz.* **72**, 1885 (1977) [*Sov. Phys. JETP* **45**, 989 (1977)].

Translated by Dave Parsons