Dipole Alfvén vortices

V. I. Petviashvili and O. A. Pokhotelov
I. V. Kurchatov Institute of Atomic Energy, Moscow; O. Yu. Shmidt Institute of Earth Physics, Academy of Sciences of the USSR

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In a plasma with a nonlinearity, kinetic Alfvén waves can propagate as two-dimensional solitary vortices.

In an Alfvén wave propagating at a large angle from the magnetic field the particle oscillation velocity becomes higher than the phase velocity, even at a low energy density, so that the wavefront may become curved, and two-dimensional traveling vortices may form. A description of this effect requires examination of nonlinear equations incorporating the dispersion and inhomogeneity of the plasma. Mikhaĭlovskiĭ *et al.*[1] have derived such equations by assuming a steady-state wave and by ignoring the ion pressure. This simplification is justified only in a highly nonisothermal plasma. The solutions found for these equations in Ref. 2 do not join in the proper way and are thus incorrect. A simple system of evolutionary equations was derived in Ref. 3 for long (in comparison with the ion Larmor radius) Alfvén and flute waves in a low-pressure plasma. In dimensionless form, this system of equations is

$$\frac{d}{dt}\Delta\phi + \mathrm{div}\{P_i, \nabla_\perp\phi\} = \{A, J\} - \frac{\partial J}{\partial z}; \quad J \equiv \Delta A; \tag{1}$$

$$\frac{dA}{dt} + \frac{\partial}{\partial z}(\phi - P_e) = \{P_e, A\}; \quad \Delta \equiv \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}, \tag{2}$$

$$\frac{dP_e}{dt} = \{A, J\} - \frac{\partial J}{\partial z}; \quad \frac{dP_i}{dt} = 0; \quad c_s/c_A \ll 1. \tag{3}$$

Here

$$\{J, A\} \equiv \frac{\partial J}{\partial x}\frac{\partial A}{\partial y} - \frac{\partial J}{\partial y}\frac{\partial A}{\partial x}; \quad \frac{d}{dt} \equiv \frac{\partial}{\partial t} + \{\phi, \ldots\};$$

$$\phi = e\varphi/T_{e0}; \quad A = eA_z\omega_{Bi}/\omega_{Pi}T_{e0};$$

$$P_i = (p_i - p_{i0})/p_{e0}; \quad P_e = (p_e - p_{e0})/p_{e0}; \quad c_s = T_{e0}/M,$$

φ is the electric potential, A_z is the component of the vector potential along the static magnetic field, the time is expressed in units of ω_{Bi}^{-1}; the coordinates x and y are expressed in units of c_s/ω_{Bi}, z is expressed in units of c_A/ω_{Bi}, c_A is the Alfvén velocity, p_{e0} is the constant part of the electron pressure, and p_{i0} is the same for the ions. The energy, $E = \int[(\nabla_\perp\phi)^2 + (\nabla_\perp A)^2 + P_e^2]d^3r$, is conserved. We seek a solution of the system of equations as a steady-state wave $\phi = \phi(x, \eta)$, $\eta = y + \alpha z - ut$, where u is the propagation velocity, and α the inclination angle. Simple transformations lead to

$$P_i = f_i(\tilde{\phi});\quad P_e = \tilde{\phi} + f_e(\tilde{A}); \tag{4}$$

$$\Delta\phi = [f_e(\tilde{A}) + f_\phi(\tilde{\phi})] / [1 + f_i'(\tilde{\phi})]; \tag{5}$$

$$J = f_A(\tilde{A}) - f_e'(\tilde{A})\tilde{\phi};\quad \tilde{\phi} \equiv \phi - ux;\quad \tilde{A} \equiv A - \alpha x, \tag{6}$$

where the functions f are arbitrary, and the prime means the derivative with respect to the argument. In the polar coordinate system $r^2 = x^2 + \eta^2$, $\tan\theta = \eta/x$, we introduce a circle of radius r_0. We choose the functions f to be different linear functions inside and outside the circle. From (5) and (6) we then find

$$\begin{gathered} \Delta\phi = B(b_\phi\tilde{\phi} + b_e\tilde{A});\quad J = b_A\tilde{A} - b_e\tilde{\phi}; \\ P_i = b_i\tilde{\phi};\quad P_e = \tilde{\phi} + b_e\tilde{A};\quad B \equiv (1 + b_i)^{-1}, \end{gathered} \tag{7}$$

where the constant coefficients b outside the circle differ from those inside the circle. Inside the circle, Eq. (7) has the solution

$$\left.\begin{aligned} \tilde{\phi} &= [e_1 J_1(k_1 r) + e_2 J_1(k_2 r)]\cos\theta; \\ \tilde{A} &= [a_1 J_1(k_1 r) + a_2 J_1(k_2 r)]\cos\theta; \end{aligned}\right\} \quad r \leqslant r_0 \tag{8}$$

Here $k_{1,2}$ are the positive roots of the equation which is the condition under which (7) has a solution in the form of the Bessel function J_1:

$$(k^2 + Bb_\phi)(k^2 + b_A) + Bb_e^2 = 0 \qquad r \leqslant r_0 \tag{9}$$

The constant coefficients e and a are chosen from the condition for a continuous transition of ϕ, A, $\partial\phi/\partial r$, $\partial A/\partial r$ into the solution outside the circle. In the case of an inhomogeneous plasma, it is a simple matter to express the external solution in terms of K_1, the modified Hankel function, by analogy with Refs. 3 and 4. Here we give a solitary solution in a homogeneous plasma. This solution must vanish at infinity; from this requirement we find a condition for the coefficients in (7) outside the circle: $b_e = -u/\alpha$, $b_A = -b_e^2$; $B = b_\phi = 1$. From (7) we then find ($\kappa^2 \equiv 1 - u^2/\alpha^2$)

$$\phi = \left[e_3 K_1(\kappa r) + \frac{e_4}{r}\right]\cos\theta;\quad A = \left[\frac{u e_3}{\alpha} K_1(\kappa r) + \frac{\alpha e_4}{ur}\right]\cos\theta;\quad r \geqslant r_0. \tag{10}$$

Furthermore, continuity at $r = r_0$ requires that we set $\tilde{\phi} = \tilde{A} = 0$. The current J and the vorticity $\Delta\phi$ are then also continuous; $P_i = 0$ and $P_e = e_3\kappa^2 K_1(\kappa r)\cos\theta$, $r \geqslant r_0$.

The two-dimensional traveling vortex found here has an arbitrary shape in the interior region, while in the exterior region, where the plasma flows around the vortex, solution (8) is determined by the velocity u and by the vortex inclination angle α. Vortices of this type may arise during the pumping of Alfvén waves in plasmas in the laboratory and in space. These vortices have apparently been observed in experiments on the artificial excitation of Alfvén pulses in the magnetosphere by means of ground-level explosions[5] and in the polar ionosphere.[6]

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