Theory of dynamic-soliton relaxation in ferromagnets

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A system of equations which describes the relaxation of dynamic solitons is proposed. The relaxation time of slow, low-amplitude solitons is calculated. Upon relaxation, the solitons undergo a self-acceleration and their magnetization precession frequency increases.

1. The nonlinear one-dimensional waves in ferromagnets have recently attracted considerable attention.[1] Two of the simplest solutions of the Landau-Lifshitz equation are the kink solitons (or the domain walls) and the dynamic solitons. The stopping of domain walls has been described phenomenologically in terms of the dissipation, using the relaxation term in the Landau-Lifshitz equation and also by analyzing the scattering of spin waves by domain walls.[2–4] Although the relaxation of dynamic solitons has virtually not been studied, the dissipation-free dynamics of these solitons is known quite well.

In this letter we consider several methods that can be used to calculate the damping of a dynamic soliton. For simplicity, we will consider a slow, one-dimensional, low-amplitude soliton in a ferromagnet. We will show that dissipative processes increase the velocity of a soliton and the frequency at which its magnetization precesses. As a result, the soliton "moves" in the ω, v^2 plane toward the interface between the region of dynamic solitons (see Fig. 1) and the spin waves and then breaks up into spin waves.

2. We work from the expression for the internal energy of a ferromagnet

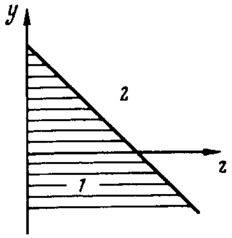

FIG. 1. 1—The region in which a dynamic soliton can be found; 2—the region in which there are spin waves. The energy of a soliton vanishes on the line $y+z=1$.

$$W=\frac{1}{2}\int\{\alpha(\partial M_i/\partial x_k)^2+\beta(M_0^2-M_z^2)\}\,d^3x \tag{1}$$

and the Landau-Lifshitz equation

$$\dot{\mathbf{M}}=\gamma[\mathbf{M},\mathbf{H}_e]+\lambda_e a^2\gamma M_0\Delta\mathbf{H}_e+(\lambda/M_0)[\mathbf{M},\dot{\mathbf{M}}]\ . \tag{2}$$

In these equations α and β are the exchange constants of the magnetic anisotropy, γ is the gyromagnetic ratio, λ_e and λ are the exchange[5] and relativistic[6] relaxation constants, a is the lattice constant, and $\mathbf{H}(\mathbf{x},t)=\delta W/\delta\mathbf{M}(\mathbf{x},t)$ is the effective magnetic field. The relaxation terms in Eq. (2) correspond to the dissipative function

$$\dot{Q}=-\frac{1}{2}\dot{W}=(\lambda M_0/4\gamma)\int[\dot{\theta}^2+\dot{\varphi}^2\sin^2\theta\,]\,d^3x+(\lambda_e a^2M_0/4\gamma)$$
$$\times\int[(\dot{\theta}_i-\varphi_i\dot{\varphi}\sin\theta\cos\theta)^2+(\dot{\varphi}_i\sin\theta\cos\theta+\dot{\theta}\varphi_i)^2+(\theta_i\dot{\varphi}+\dot{\varphi}_i\sin\theta\cos\theta)^2$$
$$+(2\varphi_i\theta_i\dot{\varphi}\dot{\theta}-\dot{\varphi}_i^2\sin^2\theta)\cos2\theta\,]\,d^3x=\dot{Q}_r+\dot{Q}_e\,, \tag{3}$$

where θ and φ are the polar and azimuthal angles of the vector M.

A dynamic soliton is characterized by the magnetization-precession frequency ω and by the center-of-mass velocity v. Instead of using these quantities, we can describe the soliton, as shown in Ref. 1, in terms of the number of spin deflections, N, occurring in it and in terms of the energy W. The number of spin deflections, N, is given by

$$\frac{\omega_0 N}{E_0}=\int(1-\cos\theta)\,d^3x=\frac{\mathcal{M}_0-\mathcal{M}_z}{\mathcal{M}_0}\,, \tag{4}$$

where $\omega_0=\gamma\beta M_0$, E_0 is the surface energy of the Bloch domain wall, $E_0=2\sqrt{\alpha\beta}\ M_0^2$, $\mathcal{M}_0=M_0V$, $\mathcal{M}_z$ is the component of the total angular momentum of the solid directed along the z axis, and V is the volume of the solid. For a dynamic soliton, the quantities N and W are[1]

$$N=(E_0/\omega_0)\,\mathrm{arsh}\,(E/E_0\Omega),$$
$$W=2E_0\sqrt{1-z-y}, \tag{5}$$

where $y = \omega/\omega_0, z = v^2/vm^2, v_m = 2\omega_0\sqrt{\alpha/\beta} = 2\omega_0 x_0$, and $\Omega^2 = 4z + y^2$.

In the absence of dissipation, E and N are the integrals of motion. Dissipation gives rise to a slow change in E and N over time. The relaxation can be described by the equations

$$\frac{1}{2}\frac{d}{dt}W^2 = -2\dot{Q}W; \quad \dot{N} = \dot{N}_{st}, \tag{6}$$

where $\dot{N}_{st}$ is the change in N due to relaxation. The dissipative function $\dot{Q}$ has already been determined. Before finding $\dot{N}_{st}$, we should point out that N is proportional (within a constant) to the relative deviation of $\mathscr{M}_z$ from the equilibrium value. The exchange relaxation therefore does not contribute to $\dot{N}_{st}$, so that this value is determined solely by the relativistic relaxation constant λ. Using (2) and (4), we find

$$\dot{N}_{st} = -\lambda(E_0/\omega_0)\int\dot{\varphi}\sin^2\theta\, d^3x. \tag{7}$$

Equations (3) and (5)–(7) constitute the total system of equations which describes the energy dissipation in a soliton, i.e., the change in ω and v over time.

3. Let us consider a low-amplitude soliton, for which $W \ll E_0$. The distribution of magnetization in a soliton of this sort is given by[1]

$$\sin\theta \approx \theta = (A/\cosh\eta); \quad \varphi = (y + 2z)\omega_0 t - z^{1/2}(x/x_0), \tag{8}$$

where $A = (W/E_0)(1+z)^{-1/2}$, and $\eta = (W/2x_0E_0)(x - vt)$.

We assume, for simplicity, that the soliton is not only of low amplitude but also slow, $v \ll v_m$. In this case, we easily see that $Q_e \ll Q_r$ and that the relaxation of W and N is described by a single parameter λ. Using distribution (8) and Eq. (3) and (7), we can easily calculate $\dot{N}_{st}$ and $\dot{Q}$ as functions of y and z. Using Eqs. (5), we can easily express $W\dot{W}$ and $\dot{N}$ in terms of $\dot{y}$ and $\dot{z}$. Finally, substituting these expressions into (6), we find a system of two differential equations of the type

$$\dot{y} = f_1(y, z); \quad \dot{z} = f_2(y, z), \tag{9}$$

whose solution yields $y = y(t)$ and $z = z(t)$.

For a slow, low-amplitude soliton we have $\Omega \approx 1$ and Eqs. (9) reduce to a single equation

$$\dot{W} = -2\lambda\omega_0 W. \tag{10}$$

It follows, therefore, that

$$W = W_0 e^{-2\lambda\omega_0 t}, \tag{11}$$

where W_0 is the initial energy of the soliton, and $\tau = (1/2\lambda\omega_0)$ is its relaxation time. Since the energy of the soliton decreases with increasing y and z (or, equivalently, with increasing ω and v^2), the energy dissipation of the soliton causes its velocity v and precession frequency ω to increase.

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