Damping of plasma waves and acceleration of resonant electrons in a transverse magnetic field

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The acceleration of electrons captured by a plasma wave is studied. This acceleration is caused by a multiple reflection of electrons from the wave front in a transverse magnetic field. The conditions under which the energy of captured electrons can be increased without restriction are determined. The inverse effect of such an acceleration on the plasma wave is analyzed and the nonlinear damping constant associated with the acceleration is calculated.

1. The idea underlying the acceleration mechanism which we study in this letter and which was suggested by one of the present authors[1] is based on the use of a magnetic field parallel to the wave front of an accelerating wave. The particles whose velocities are nearly the same as the wave velocity have a small radius of gyration and, as they are turned around by the magnetic field, they acquire the ability to be multiply reflected from the wave front. As a result, the velocity of these particles along the wave front, v_y, increases after each reflection. This velocity has a limiting value because at large values of v_y the Lorentz force $(e/c)v_yH$ is greater than the reflecting force of the electric field $-e(\partial\varphi/\partial x)$ and the particle "surmounts" the potential hump, stopping its interaction with the wave. This mechanism is responsible for the appearance of the bunches of reflected ions at the leading edge of quasitransverse shock waves, e.g., at the leading edge of a shock wave of the earth's magnetosphere.[2] For the electrons captured by a plasma-wave field, this acceleration mechanism is the main pathway of the nonlinear damping of the plasma wave in a transverse magnetic field.[3] The device for accelerating charged particles captured by a plasma wave, which was proposed by Dawson[4] and which has been termed a serfotron, is essentially a relativistic modification of the mechanism for the acceleration of captured particles and the concomitant plasma-wave dissipation in the transverse magnetic field, which was studied by Sagdeev and Shapiro.[3] Because of the relativistic restriction of the velocity v_y, the Lorentz force $(e/c)v_yH$ of a relativistic particle in a weak magnetic field is always smaller than the electric force $-e(\partial\varphi/\partial x)$, and the particle, being trapped in the potential well, can increase its energy without restriction.

2. The system of equations which describes the interaction of an electron with a traveling plasma wave $E_x = -\partial\varphi/\partial x$, $\varphi = \varphi_0\cos(kx - \omega t)$ in a transverse magnetic field $H_0 \| 0z$ can be written

$$dp'_x/dt' = e\partial\varphi'/\partial x' - \frac{eH'_0}{c}v'_y, \tag{1}$$

$$dp'_y/dt' = e\beta_\phi H'_0 + \frac{eH'_0}{c}v'_x. \tag{2}$$

Here and elsewhere in the text, the primes are used to denote quantities referring to the coordinate system of the wave. In this coordinate system, the presence of an electric field parallel to the wave front, $E'_y = -\beta_\phi H'_0, \beta_\phi = \omega/kc$, is taken into account in Eq. (2). For a particle captured by the field of a small-amplitude wave, we have $v'_x \ll \omega/k$, and we can infer from Eq. (2) that the component of the momentum directed along the wave front, p_y, increases approximately linearly over time.[3] Taking the small $\sim v'_x/c$ into account and transforming to a laboratory coordinate system, we can write the following equation for $v_y(t)$:

$$v_y(t) = \frac{\omega_n \beta_\phi t \sqrt{1-\beta_\phi^2}}{\sqrt{1+\omega_n^2\beta_\phi^2 t^2}} \left[1 - \beta_\phi \frac{v'_x}{c} + \frac{1-\beta_\phi^2}{1+\beta_\phi^2\omega_n^2 t^2} \frac{1}{\beta_\phi t} \int_0^t \frac{v'_x}{c} dt \right], \quad (3)$$

where $\omega_n = eH_0/mc$ is the nonrelativistic cyclotron frequency. In addition to being accelerated along the wave front, the particle executes phase oscillations relative to the wave that captured it. To analyze these oscillations, we will use the energy integral of the original equations [Eqs. (1) and (2)]

$$mc^2 \left[\frac{1}{\sqrt{1 - \frac{v'^2_x}{c^2} - \frac{v'^2_y}{c^2}}} - 1 \right] - e\varphi'(x') - \frac{e\beta_\phi}{\sqrt{1-\beta_\phi^2}} H_0 \int_0^t v_y dt = \mathcal{E}. \quad (4)$$

We assume that the following conditions hold:

$$e\varphi' \ll mc^2, \qquad \omega_n \tau_0 \ll 1. \quad (5)$$

Here $\tau_0 \sim 1/k'v'_x$ is the period of the phase oscillations of the captured particle. From (4), with the help of (3), we then find the following equations for the reversal points of the captured particle, $x'_\pm$, and where $v'_x = 0$:

$$\mathcal{E} + e\varphi'(x'_\pm) = \frac{eH_0}{c} \left[v_y(t)(x'_\pm(t) - x'(0)) + \beta_\phi^2 \sqrt{1-\beta_\phi^2} \int_0^t v_y(t) \left[\frac{1}{1-\beta_\phi^2} - \frac{3}{(1+\omega_n^2\beta_\phi^2 t^2)^2} \right] v'_x dt \right]. \quad (6)$$

We can infer from this equation that the magnetic field increases the effective energy of the phase oscillations when the particle moves toward the left reversal point x'_- and decreases the effective energy of the phase oscillations when the particle moves in the opposite direction, i.e., toward the right reversal point x'_+, moving the particle trajectory to the left. Energy integral (4) and the condition under which the longitudinal adiabatic invariant is conserved imply that the amplitude of the phase oscillations, $\Delta x'$, of a relativistic particle does not change over time and that the frequency $v'_x/\Delta x'$ decreases as $1/t$. Accordingly, we can write in an approximate manner the law governing the temporal variation of the reversal points

$$x'_{\pm} = \pm x'_0 - \xi(t), \tag{7}$$

where $\pm x'_0$ are the reversal points in the absence of a magnetic field. These points are related to the particle energy by $\mathscr{E} = -e\varphi'_0 \cos kx_0$, and the equation for $\xi(t)$, which can be derived from (6) with the help of the second condition in (5), is

$$\frac{E_0}{H_0}\,\frac{\sin kx_0\,\sin k'\xi}{kx_0} = \frac{\beta_\phi\,\omega_n t}{\sqrt{(1-\beta_\phi^2)(1+\beta_\phi^2\omega_n^2 t^2)}} \times [1 - 3\beta_\phi^2(1-\beta_\phi^2)(1+\beta_\phi^2\omega_n^2 t^2)^{-1}]\,. \tag{8}$$

In the nonrelativistic limit $E_0/H_0 \ll 1$, $\beta_\phi \ll 1$, this equation is the same as that found in Ref. 3.

For a particle with the initial reversal points $kx_0 < \pi/2$ to remain in the potential well in the limit $t \to \infty$, the condition

$$\frac{E_0}{H_0}\,\frac{\sin kx_0}{kx_0} > \frac{1}{\sqrt{1-\beta_\phi^2}} \tag{9}$$

must be satisfied [as we can see from (6), in this case the condition $k'\xi < \pi/2$ always holds]. At the time the particle with $kx_0 > \pi/2$ leaves the well, we have $\xi = \pi/k' - x'_0$, and the condition under which the particle is trapped is

$$\frac{E_0}{H_0}\,\frac{\sin^2 kx_0}{kx_0} > \frac{1}{\sqrt{1-\beta_\phi^2}}\,. \tag{10}$$

Because of the approximate nature of the analytic solution, whose range of applicability is limited[1)] by conditions (5), we will integrate numerically the original system of equations [Eqs. (1) and (2)]. The results of integration for different values of the parameters E_0/H_0, $\kappa = (\cos k)/\omega_n$, and β_ϕ are shown in Figs. 1–3. Figure 1 shows a typical trajectory of the particle which is drawn into a process of unlimited acceleration. The magnetic field displaces both reversal points of the trapped particle to the left, and the particle remains in the potential well an arbitrarily long time, oscillating at an approximately constant amplitude and frequency that attenuate over time. Such an unbounded capture of particles by a wave can occur at reasonably large values of the parameter $E_0/H_0 > [E_0/H_0]_{\text{thr}}$. The dependence of $[E_0/H_0]_{\text{thr}}$ on kx_0 found by means of a numerical simulation at $\kappa = 5$ is shown in Fig. 2. In this figure the dashed curve is the analytic dependence found from relations (9) and (10). The particles that

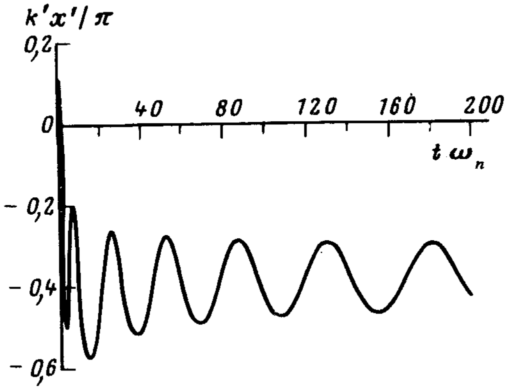

FIG. 1. Typical trajectory $k'x'(t)$ of the phase oscillations of an accelerated electron, found for $E_0/H_0 = 1.3$. In Figs. 1–3 $\kappa = 5$ and $\beta_\phi = 0.4$.

have entered a region of indefinitely long capture form an accelerated bunch. The energy of these particles increases with time in accordance with the equation

$$W = mc^2 \left[\frac{1 + \beta_\phi^2 \omega_n^2 t^2}{1 - \beta_n^2} \right]^{1/2} . \tag{11}$$

An increase of this sort will eventually correspond to the first term in Eq. (3) for $v_y(t)$. The remaining terms give rise to a small spread of energies in the accelerated bunch, whose relative amplitude $\Delta W/W \sim v_x'/c$ eventually decreases as $1/t$.

The functional dependence $W(t)$ obtained numerically is nearly linear (see Fig. 3). Figure 3 also shows the energy spectrum of a bunch of captured particles at $\omega_n t = 100$. The phases of the accelerated particles vary within the range $-\pi/3 < k'x'\pi/6$ and the energy spread of these particles, $\Delta W/W$, is no greater than 4%. These results show that the acceleration method considered by us is highly effective.

The mechanism responsible for the acceleration of electrons captured by a wave is the principal cause of nonlinear damping of a plasma wave in a weak, transverse magnetic field. If we disregard the escape of resonant particles from the potential well, which, according to (9) and (10), can be done when $E_0/H_0 \gg 1$ the law governing the damping of the plasma-wave amplitude, which is found from the condition for the energy conservation in the wave-resonant particles system, will have an explosive nature

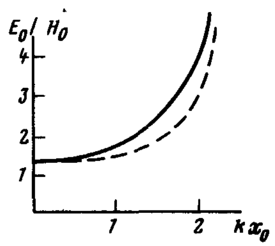

FIG. 2. The plot of $[E_0/H_0]_{\rm thr}$ versus kx_0. The values of E_0/H_0 above the threshold value correspond to an unlimited acceleration of electrons. The solid curve is the result of a numerical simulation and the dashed curve is the analytical dependence found from Eqs. (9) and (10).

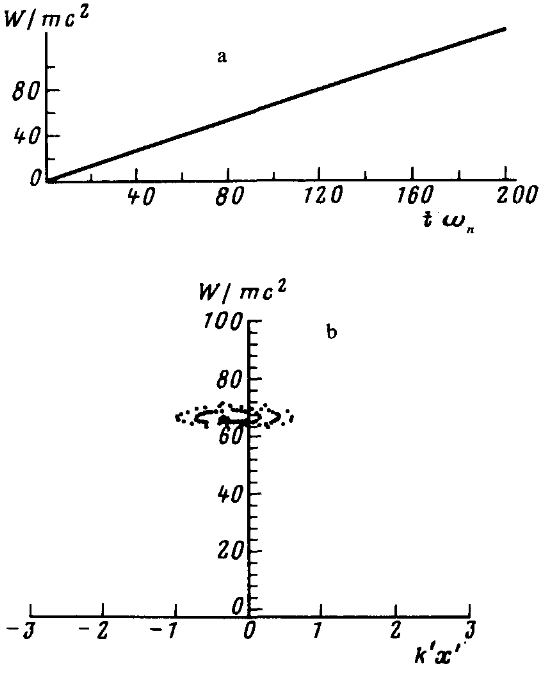

FIG. 3. The time dependence of the energy of an accelerated particle for $E_0/H_0 = 1.3$ and $kx_0 = 0.45$; (b) dependence of the energy W of a particle on its phase $k'x'$ at $\omega_n t = 100$ and $E_0/H_0 = 4.0$.

$$E_0(t)=E_0(0)\left[1-\frac{t^2}{t_0^2}\right]^{2/3}, \quad t_0^2=\frac{\pi}{3}\frac{\Omega_0^3}{\gamma_L\omega^2\omega_n^2}\frac{v_\phi^2}{v_T^2\sqrt{1-\beta_\phi^2}}. \qquad (12)$$

Here $\Omega_0=\sqrt{eE(0)k/m}$ is the nonrelativistic bounce frequency, v_{th} is the thermal velocity, and

$$\gamma_L=\sqrt{\frac{\pi}{8}}\,\omega\,\frac{v_\phi^3}{v_{th}^3}\exp\left[-\frac{mc^2}{T}\left(\frac{1}{\sqrt{1-\beta_\phi^2}}-1\right)\right]$$

is the Landau damping constant for a plasma wave with $\beta_\phi \sim 1$ (see Ref. 5). In deriving this equation, we assumed, for simplicity, that $\beta_\phi\omega_n t_0 \ll 1$. From (11) we find that if the condition

$$\Omega_0^{3/2}\gamma_L^{1/2} \gg \omega_n k v_{th}(1-\beta_\phi^2)^{1/4}$$

is satisfied, the damping time of the wave will be $t \ll \gamma_L^{-1}$; i.e., the damping time of the wave will be even shorter than the linear damping in the absence of a magnetic field.

[1)]The parameter $E_0/H_0=(e\varphi'/mc^2)(k'c/\omega_n)$, which characterizes the possibility of capturing the particles as they are accelerated in an unrestricted manner, may be rather large if conditions (5) hold.

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