Interface dynamics in first-order phase transitions

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The dynamics of the interface which arises in a first-order spin-flip phase transition in an antiferromagnet is analyzed. The limiting velocity of the interface is found as a function of the external agents acting on the system.

In most studies of nonlinear phenomena in the kinetics of first-order phase transitions of the order-disorder type, the thermodynamic potential of the system has been expanded in powers of the order parameter and its derivatives (e.g., Ref. 1). The evolution of the order parameter is described by the Landau-Khalatnikov equation,[2] which has a kink solution corresponding to an interface in uniform motion. This approach is justified only if the order parameter and its gradients are small, i.e., only if the first-order phase transition that occurs is approximately a second-order transition.

In this letter we examine the nonlinear dynamics of an interface which arises in a thermally induced spin flip in an antiferromagnet. For definiteness, we assume that the magnetic phases participating in the first-order spin-flip phase transition differ by an angle of $\pi/2$ in the orientation of the antiferromagnetic vector $\mathbf{l}$. Obviously, the approach outlined above cannot be used in this case, and we instead use the Landau-Lifshitz dynamic equations for the magnetization to describe the motion of the interface.

There are two important points to be noted. First, there is generally an abrupt change in the entropy of the system at a first-order phase transition. For this reason, the heat-balance equation in the system must be solved along with the evolution equation for the order parameter. In the case under consideration, however, of a spin-flip phase transition, the jump in the entropy is small, to the extent that the relativistic interactions are weak in comparison with the exchange interactions,[1)] and this jump can be ignored. Second, during first-order phase transitions, which are approximately of second order, it is necessary to take into account the random force caused by fluctuations in the system in the evolution equation for the order parameter.[3] In a first-order spin-flip phase transition, the interval over which the different magnetic phases coexist is quite large (several degrees or even tens of degrees); i.e., there is a broad temperature interval in which the Ginzburg-Levanyuk condition holds, and in which fluctuations can be ignored.

We will accordingly describe the motion of the interface by the Landau-Lifshitz equations, taking the relaxation term into account in the Hilbert form. As was shown in Refs. 4 and 5, in a two-sublattice antiferromagnet these equations reduce to a single effective equation of motion for the angle (θ) made by the antiferromagnetic vector **l** with a selected axis in the crystal:

$$\alpha\left(\frac{\partial^2\theta}{\partial\xi^2}-\frac{1}{c^2}\frac{\partial^2\theta}{\partial t^2}\right)-A\sin\theta\cos\theta-B\sin 2\theta\cos 2\theta=\lambda\frac{\partial\theta}{\partial t}\ . \tag{1}$$

Here $c=(1/2)\,gM_0(\alpha\delta)^{1/2}$ is the minimum phase velocity of the spin waves, M_0 is the saturation magnetization of the sublattice of the antiferromagnet, g is the gyromagnetic ratio, α and δ are the constants of the inhomogeneous and homogeneous exchange interactions, respectively, A and B are combinations of phenomenological constants of the antiferromagnet, which are generally functions of the temperature and the external fields, λ is the Hilbert damping parameter; and ξ is the coordinate along which the magnetic inhomogeneity exists (the interface is assumed planar).

Steady-state homogeneous solutions of Eq. (1) with $B>0$ correspond to equilibrium magnetic states[2)] $\phi_1(\theta=0)$ and $\phi_2(\theta=\pi/2)$.[2)] The regions of stability of the ϕ_1 and ϕ_2 phases ($A\geqslant-2B$ and $A\leqslant 2B$, respectively) overlap, and at the point (or on the line) defined by the condition $A=0$ the first-order spin-flip phase transition $\phi_1\rightleftarrows\phi_2$ occurs.

With $A>0$ in (1), with the boundary conditions $\theta(\xi\to\pm\infty)=0$ or π and $(\partial\theta/\partial\xi)(\xi\to\pm\infty)=0$, which correspond to the phase ϕ_1, we have the steady-state solution

$$\tan\theta=\sqrt{\frac{A+2B}{A}}\sinh^{-1}\left(\xi\sqrt{(A+2B)/\alpha}\right), \tag{2}$$

which describes a 180° domain wall in the ϕ_1 phase. In the region in which the ϕ_1 phase is metastable ($-2B\leqslant A\leqslant 0$), Eq. (1), with the same boundary conditions, has a soliton steady-state solution,[5] which is unstable.

Analogous solutions hold for the ϕ_2 phase, in which we have $\theta(\xi\to\pm\infty)=\pm\pi/2$, $(\partial\theta/\partial\xi)(\xi\to\pm\infty)=0$.

Finally, at $A=0$, i.e., at the point of the first-order phase transition, the steady-state solution of the equation describes a 90° domain wall (an interface between the ϕ_1 and ϕ_2 phases):

$$\tan\theta=\exp(-\xi/\xi_0),\quad \xi_0=\sqrt{\alpha/2B}\ , \tag{3}$$

where ξ_0 is the effective thickness of the interface. We wish to emphasize that an interface at rest exists only at the point of phase equilibrium ($A=0$).

Upon a deviation of the external parameters, an existing interface begins to move away from this point in such a way that the thermodynamically favored phase increases in size. This situation occurs, for example, in the growth of nucleating regions in a first-order phase transition (treating the interface as one-dimensional is legitimate if the radius of curvature, R, of the transition layer is sufficiently large: $R\gg\xi_0$) and upon a deviation from phase equilibrium in a two-phase system, which is of the nature of a thermodynamically stable, stripe, transitional domain structure (an intermediate state).[7,8]

A solution of Eq. (1) corresponding to a 90° interface in uniform motion (a plane phase-transition front) is

$$\tan\theta = \exp\left(-\frac{\xi - v\,t}{\xi_0\sqrt{1-(v/c)^2}}\right). \tag{4}$$

The velocity (v) of this interface is determined by the balance between the "pressure force," i.e., a measure of the deviation of the system from the phase-equilibrium point, and the "friction force" $\lambda\partial\theta/\partial t$:

$$v = \frac{\mu\,|A\,|}{\sqrt{1+(\mu A/c\,)^2}}, \qquad \mu = \xi_0/\lambda. \tag{5}$$

In contrast with the problem under consideration here, the velocity of the interface in an ordering phase transition is determined by the rate of heat transfer in the system.

It follows from (5) that the velocity of the interface (like that of a 180° domain wall[4,5]) does not exceed the minimum phase velocity of the spin waves, c. The highest possible velocity of the interface, v_m, obviously is reached near the boundaries of the region of metastability, i.e., as $|A\,|\to 2B$. Substituting the characteristic values of the parameters for antiferromagnets of the rare-earth orthoferrite type ($\delta\sim 10^4$, $c\sim 10^5$ cm/s, $\lambda g M_0\sim 10^{-4}$, and $B\sim 10^{-1}$) for estimates, we find that v_m can formally reach values close to the limiting velocity c. Accordingly, in addition to the need to consider the fluctuations in the case $|A\,|\to 2B$, mentioned earlier, we have yet another restriction, which arises because the macroscopic description of the interface cannot be used in the limit $v\to c$ [here the effective thickness of the interface, $\xi_0\sqrt{1-(v/c)^2}$, becomes comparable to the lattice constant].

[1)]In a first-order phase transition of the magnetic-ordering type, the jump in the entropy is of exchange origin and cannot be ignored.

[2)]The interfaces between phases with different equilibrium angles can be treated in a similar way. The motion of a 90° interface in a spin-flip phase transition was studied in Ref. 6.

[1]A. Cordon, Phys. Lett. **99A**, 139 (1983).

[2]L. D. Landau and I. M. Khalatnikov, L. D. Landau, Sbornik trudov (L. D. Landau Collection), Vol. 2, Nauka, Moscow, 1969, p. 218.

[3]A. Z. Patashinskiĭ and B. I. Shumilo, Zh. Eksp. Teor. Fiz. **77**, 1417 (1979) [Sov. Phys. JETP **50**, 712 (1979)].

[4]I. V. Bar'yakhtar and B. A. Ivanov, Fiz. Nizk. Temp. **5**, 759 (1979) [Sov. J. Low Temp. Phys. **5**, 361 (1979)].

[5]V. G. Bar'yakhtar, B. A. Ivanov, and A. L. Sukstanskiĭ, Zh. Eksp. Teor. Fiz. **78**, 1509 (1980) [Sov. Phys. JETP **51**, 759 (1980)].

[6]B. A. Ivanov, Zh. Eksp. Teor. Fiz. **79**, 581 (1980) [Sov. Phys. JETP **52**, 293 (1980)].

[7]V. G. Bar'yakhtar, A. E. Borovik, and V. A. Popov, Pis'ma Zh. Eksp. Teor. Fiz. **9**, 634 (1969) [JETP Lett. **9**, 391 (1969)].

[8]K. L. Dudko, V. V. Eremenko, and V. M. Fridman, Zh. Eksp. Teor. Fiz. **61**, 678 (1971) [Sov. Phys. JETP **34**, 362 (1971)].

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