Impurities and electron-electron interaction in the quantum Hall effect

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A model for short-range electrons and impurities is used to explain the integral and fractional quantization of the Hall effect.

Integral quantization of the Hall effect, which was experimentally detected by von Klitzing *et al.*,[1] can ordinarily be explained by a theory which takes into account only the interaction of electrons with impurities.[2–8] Fractional quantization of the Hall effect, which was discovered by Tsui *et al.*,[9] cannot be explained without taking the interaction of electrons into account. The theory proposed by Laughlin[10] and developed by Haldane[11] and Halperin[12] considers only the electron-electron interaction. It is clear that the interaction with the impurities is appreciable in the case of fractional quantization. Specifically, this interaction accounts for the steps on the curve of σ_{xy} versus the magnetic field. The accuracy of integral quantization is so high that possible corrections to the single-electron theory become important. Niu and Thouless[13] and Avron and Seiler[14] have shown that integral quantization of the Hall conductivity is topological in nature and that it can be realized if the ground state of the electron system is nondegenerate and is separated from the excited states by an energy gap. The analysis of these investigators is not restricted to a specific model, but they have failed to inquire what determines the desirable properties of the ground state.

We propose a simple model which takes into account the interaction between the electrons and the interaction of the electrons with the impurities. In this model the Hall conductivity remains constant over a certain interval of magnetic fields or electron densities and is equal to the integral number of photons e^2/h or to a photon which is divided by an odd number m.

In this model the Coulomb interaction between the particles is replaced by a short-range repulsive potential $V(r)$. The interaction of electrons with impurities $V_i(r)$ is also assumed to be a short-range repulsive potential. Our task is to find the ground state for the given total number of particles, N, for the given impurities, N_i, and their arrangement which is assumed random, and also for the given external magnetic field which is characterized by the quanta of magnetic flux, $2S$, across the area occupied by the system.

We will first consider a simple, impurity-free problem which was solved previously by Pokrovskiĭ *et al.*[15] and Trugman and Kivelson.[16] In leading order in the small parameter $(a/l_H)^2$, where a is the interaction radius, and l_H is the magnetic length, any wave function which has at least a third-order zero generates a zero-point energy if the coordinates of the electron pair are the same. Such functions are valid only if $\nu \equiv N/2S \leqslant 1/3$, and the ground state can be realized at such values of ν. At $\nu = 1/3$ the wave function is the same as the function ψ_L, which was used by Laughlin[10] for $m = 3$ (the notation is the same as in Ref. 10).

$$\psi_l = \prod_{1 \leqslant i < k \leqslant N} (z_i - z_k)^m e^{-\Sigma |z_j|^2/4} . \tag{1}$$

Laughlin's ground state is nondegenerate. The lowest excited state with $\nu = 1/3$ has a finite energy of order $V_1 \sim \int V(r) r^2 d^2 r$. At $\nu < 1/3$ the ground state is multiply degenerate. The wave function of any of these states is

$$\psi = \psi_L \cdot Q(z_1, \ldots, z_N), \tag{2}$$

where Q is a symmetric polynomial of order no higher than $s = 2S - 3(N-1)$ in each variable.

We in fact represent $V(r)$ as a series

$$V(r) = V_0 \,\delta(\mathbf{r}) + V_1 \,\Delta\,\delta(\mathbf{r}) + V_2 \Delta^2 \,\delta(\mathbf{r}) + \ldots . \tag{3}$$

So far we have restricted the analysis to the first two terms of this series. By including the next two terms we can reduce the ground-state degeneracy for $1/5 \leqslant \nu \leqslant 1/3$. For $\nu = 1/5$ the state described by the wave function ψ_L with $m = 5$ becomes a nondegenerate ground state which is separated from any excitation by a gap whose width is $V_3 \sim \int V(r) r^6 d^2 r$. We thus have a sequence of ground states with $\nu = 1/m$ which are separated from the excited states by gaps. The ground-state energy of the system of interacting electrons as a function of ν has structural features at $\nu = 1/m$. The interaction betwen electrons cannot change σ_{xy}, which, as in the case of free electrons, is Nec/H.

Let us now consider a system with impurities. We expand the potential of interaction of an electron with an impurity in a series in the derivatives of the δ functions and we retain only the first term:

$$V_i(r) = V_i \,\delta(\mathbf{r}) . \tag{4}$$

We assume that $\nu < 1/3$. The ground state in this case is multiply degenerate in the absence of impurities. The impurities lift the degeneracy. Since V_i is positive, it would be desirable that the wave function vanish at the points where the impurities are located. We thus obtain the following ground-state function:

$$\psi = \prod_{\substack{1 \leqslant j \leqslant N \\ 1 \leqslant k \leqslant N_i}} (z_j - \zeta_k)\, \psi_L(z_1, \ldots, z_N), \tag{5}$$

where $\zeta_k = x_k + iy_k$ is a complex coordinate of the k-th impurity. The energy of this state is zero in first approximation in $(a/l_H)^2$. A similar situation in single-electron approximation was reported by Baskin *et al.*[17] Such a state can, however, be realized only if the condition $N_i \leqslant s = 2S - 3(N-1)$ is satisfied. In the case of an absolute inequality, the ground state remains degenerate. The wave function (5) is amenable to a graphic interpretation: A Laughlin's quasihole[10] with a charge $+|e|/3$ is trapped at each impurity.

At $N_i > s$ the quasiholes can no longer be distributed among all impurities. We seek the ground-state wave function in the form like that in (5)

$$\psi = \prod_{\substack{1 \leqslant j \leqslant N \\ 1 \leqslant k \leqslant s}} (z_j - \zeta_k)\, \psi_L(z_1, \ldots, z_N), \tag{6}$$

where the s impurities are chosen from N_i with the stipulation that the energy be minimum. The ground-state wave function like that in (6) can be obtained if the conditions $V_i < V_1$ and $N_i \ll N$ are satisfied. The ground-state energy is

$$E_0 = V_i \sum_{s < k \leqslant N_i} \rho(\zeta_k), \tag{7}$$

where $\rho(\zeta)$ is the electron density at the point ζ. Far from the holes, the density $\rho(\zeta)$ is nearly independent of the coordinates and is approximately equal to $N/2S$. The weak density fluctuations, however, determine the best choice of the impurities on which the holes are trapped.

The uniform electric field $\mathbf{E}$ causes the electrons to drift. We transform to a coordinate system that moves at a velocity $v_D = c[(\mathbf{E}\times\mathbf{H})/H^2]$. Since the electric field is zero in this coordinate system, the electrons that produce Laughlin's liquid are stationary and the quasiholes associated with the impurities move at a velocity $\mathbf{v}_D$. The quasiholes with a charge $|e|/3$ account for the following contribution to the current $\mathbf{j}$:

$$\mathbf{j}_h = -\mathbf{v}_D \frac{|e|}{3} s\,. \tag{8}$$

In the laboratory coordinate system the current is

$$\mathbf{j} = e\mathbf{v}_D N - \mathbf{v}_D \frac{|e|}{3} s = e\mathbf{v}_D \left(N + \frac{s}{3}\right). \tag{9}$$

From the definition $s = 2S - 3N$ it is clear that the current j does not depend on the electrons or impurities and is

$$\mathbf{j} = -\frac{1}{3}\,\frac{e^2}{h}\,\frac{\mathbf{E}\times\mathbf{H}}{H}, \tag{10}$$

which is equivalent to the condition under which σ_{xy} is quantized at least in the magnetic-field interval $3N < 2S < 3N + N_i$ or the density interval $(1/3) - (N_i/2S) < \nu < (1/3)$. The steps near $\nu = 1/m$, and, in particular, at $m = 1$ arise in a similar way, which corresponds to the integral quantization of the Hall effect. The quantization remains in effect in a certain region of the parameter (a/l_H).

Let us determine the relationship between our approach and the Niu-Thouless-Avron-Seiler theorem. The ground state with energy (7) is evidently nondegenerate. In a large system, the nearest excited states are nonetheless separated from the ground state by an energy interval inversely proportional to the size of the system, forming a nearly continuous spectrum. To reach the nearest excited states, however, it is necessary to interchange the quasiholes from many impurities. This means that the ground state lags far behind the first excited state in functional space. We believe that this condition is adequate for a topological quantization of the Hall conductivity.

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