Decay of optical solitons

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Numerical experiments on nonlinear dynamics of subpicosecond pulses in optical fibers have shown that the bound states of solitons are structurally unstable: The multisoliton pulses decay, setting up envelope shock waves.

One of the recent major achievements of nonlinear optics has been the experimental realization of optical solitons in the envelope of optical fibers.[1] The main branches of optical-soliton research have now been clearly defined. First, this work involves the development of nonlinear fiber-optic communication lines carrying large bulks of information and, secondly, it involves the development of new methods of generating extremely short pulses which make use of the soliton phenomena in optical fibers, and also the development of new types of lasers—"soliton lasers."

The dynamics of optical solitons has so far been described in terms of the nonlinear Schrödinger equation for a complex wave-packet envelope.[2] The use of this model makes it possible to correctly describe the experimental results down to a picosecond range of the optical-pulse envelope.[1,3] The recent generation of femtosecond pulses[4,5] has created the need for generalizing the theory of nonlinear waves to this frequency range.

As we move into the femtosecond region, the use of small terms of higher order in the parameter $\alpha = T/\tau_v$ (where τ_v is the scale of variation of the complex wave-packet envelope as a function of time, and T is the period of light oscillations) in the exact equation for a complex wave-packet envelope in a nonlinear dispersive medium is of fundamental importance.[6]

In this letter we report the results of numerical experimental studies of the nonlinear dynamics of intense femtosecond pulses in optical fibers. We found the multisoliton pulses to be structurally unstable and we determined the range of the principal parameters of the problem within which the solitary wave packets with the envelope duration $\tau_v \lesssim 10$ T can be generated. The aim of this study is to provide an incentive for experimental studies in the field of nonlinear femtosecond optics.

We will analyze the evolution of the initial wave packet in terms of the modified nonlinear Schrödinger equation

$$i\,\partial\Psi/\partial z = \frac{1}{2}\,\partial^2\Psi/\partial\tau^2 + R\,|\Psi|^2\,\Psi - i\gamma\,\frac{\partial}{\partial\tau}\,(\,|\Psi|^2\,\Psi\,). \tag{1}$$

Here we have introduced the following dimensionless parameters:

$$\Psi = E/E_0;\ \ \tau = \frac{t - z/v}{\tau_0};\ \ z = z/z_{disp};\ \ \ z_{disp} = \frac{\tau^2_0}{|\,\partial^2 K/\partial\omega^2\,|};$$

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$$R = N^2 = \tilde{\alpha}\,\frac{z_{disp}}{z_{nl}}\,; \quad z_{nl} = \frac{1}{k n_2 E_0^2}\,; \quad \gamma = R\,\frac{2n_0}{c\tau_0 k}\,,$$

where $\tilde{\alpha}$ is the geometric factor which arises as a result of averaging the nonlinear polarization over the transverse cross section of the optical fiber, N is the number of solitons per pulse, and E_0 and τ_0 are the initial amplitude and the pulse length; the perturbing term in Eq. (1) describes the onset of the envelope shock waves.[7–9]

Multiplying (1) first by Ψ^* and then by Ψ^*_τ and combining it with a complex-conjugate equation, we easily find

$$\partial\rho / \partial z = \frac{1}{2}\,\frac{\partial}{\partial\tau}\,[p - 3\gamma\rho^2]\,, \tag{2}$$

$$\partial p / \partial z = -2\gamma\,[\,\rho\,\mathrm{Im}\,\Psi_\tau^2 - \frac{1}{2}\,\rho\, p_\tau\,] + 2\,\frac{\partial}{\partial\tau}\,[\,-H - \frac{1}{2}\,\mathrm{Re}\,\Psi^*\Psi_{\tau\tau} - \frac{3}{2}\,\gamma\rho p\,]\,,$$

$$\partial H/\partial z = \frac{3}{2}\,\gamma\,|\,\Psi_\tau\,|^2\,\rho_\tau$$

$$+\,\frac{1}{2}\,\frac{\partial}{\partial\tau}\,[\,R\,\rho\, p + \gamma\,\rho\,|\,\Psi_\tau\,|^2 + \frac{1}{2}\,\gamma\,(\rho_\tau)^2 + R\gamma\rho^3 + 2.\mathrm{Im}\,\Psi_\tau^*\Psi_{\tau\tau}\,].$$

where

$$\rho = |\,\Psi\,|^2; \quad p = 2\,\mathrm{Im}\,\Psi^*\,\Psi_\tau\,; \qquad H = -\frac{1}{2}\,|\,\Psi_\tau\,|^2 + \frac{R}{2}\,|\,\Psi\,|^4\,.$$

Integrating (2) over τ, we find that only the first integral of motion of the unperturbed nonlinear Schrödinger equation is retained in the model under consideration:

$$\frac{\partial}{\partial z}\,\left(\int |\,\Psi\,|^2\, d\tau\right) = 0\,.$$

Numerical studies have shown that a perturbation in nonlinear Schrödinger equation (1) gives rise to very important, qualitatively new systematic features in the nonlinear dynamics of intense subpicosecond pulses.

The propagation of a single-soliton pulse [of the fundamental soliton of the nonlinear Schrödinger equation, $\Psi(0,\tau) = \mathrm{sech}(\tau)$] is accompanied by a retardation of the envelope peak and by a considerable frequency modulation of the pulse (Fig. 1).

The propagation of a multisoliton pulse in an optical fiber at $\tau_v \gg T$ in the region of negative dispersion of the group velocities, $\partial^2 k/\partial\omega^2 < 0$, is accompanied by a periodic modulation of the envelope: a self-steepening of the pulse, followed by a formation of a narrow central peak against the background of a broad pedestal, its division into separate fragments, and a subsequent return to the original shape.[1,3,4]

A study of the dynamics of the bound states of solitons in terms of modified nonlinear Schrödinger equation (1) has shown that the perturbing term accounts for

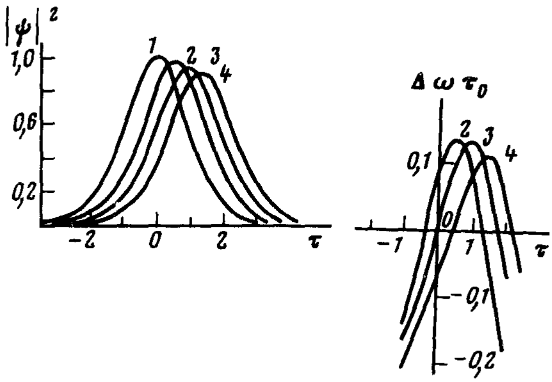

FIG. 1. Nonlinear dynamics of a single-soliton pulse $\psi(z=0,\tau)=\text{sech}(\tau)$ $(R=1)$ at $\gamma=0.25$. a—The shape of the envelope at $z=0$ (1), at $z=z_0$ (2), at $z=2z_0$ (3), and at $z=3z_0$ (4), where $z_0=(\pi/2)z_{\text{disp}}$ is the period of the soliton; b—frequency modulation of the pulse for the same parameters of the problem.

the instability of the bound state of solitons in the nonlinear Schrödinger equation and for its decay. We wish to emphasize that Zakharov and Shabat[2] have indicated that the bound states of solitons can become unstable in the unperturbed nonlinear Schrödinger equation $(\gamma\equiv0)$. Numerical experiments based on the nonlinear Schrödinger equation $(\gamma\equiv0)$ and physical experiments with optical solitons $(\tau_v \gg T)$ have shown, however, that multisoliton pulses are long-lived, highly stable pulses.[3,10] Makhan'kov attributed this finding to retention of the higher integrals of motion in the nonlinear Schrödinger equation at $\gamma\equiv0$. As we move into the femtosecond region, beyond $T/$

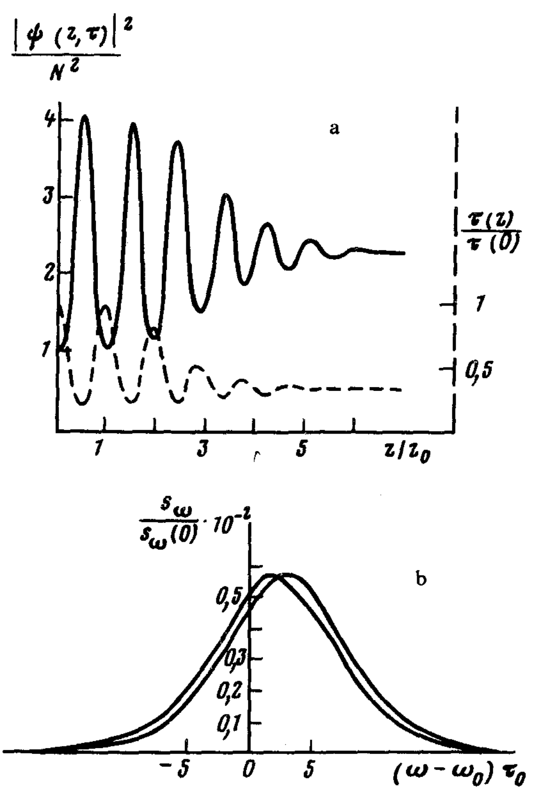

FIG. 2. Dynamics of the decay of the bound state of two solitons $(R=4)$ at $\gamma=0.1$. a—The intensity and length of the pulse produced due to the decay versus the length z; b—the spectrum of the intense pulse at $z=7z_0$ (1) and at $z=10z_0$ (2).

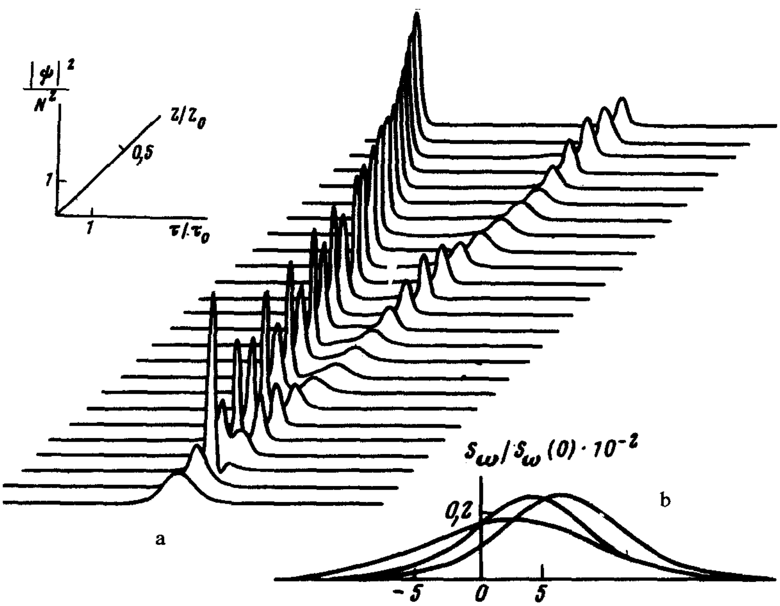

FIG. 3. a—Dynamics of the decay of the bound state of three solitons ($R = 9$) at $\gamma = 0.1$; (b)—the spectrum of the intense pulse at $z = z_0$ (1), at $z = 2z_0$ (2), and at $z = 3z_0$ (3).

$\tau_0 \simeq 0.01$ the physical picture of the propagation of a multisoliton pulse in an optical fiber changes dramatically. The lifetime of the bound state of solitons decreases significantly, principally because of the preferential enrichment of the rf region of the spectrum which is described by the perturbing term in nonlinear Schrödinger equation (1). A periodic change in the sign of the frequency modulation due to the propagation of a multisoliton pulse accounts for the generation of an intense narrow pulse near the self-steepening point at the leading edge of the envelope. The center of gravity of the spectrum of the generated pulse shifts continuously toward the high-frequency region as the pulse propagates. Figure 2 shows the dynamics of the decay of the initial packet $\Psi(z = 0,\tau) = 2\ \mathrm{sech}\tau$, which was calculated by us. Figure 2 also shows the dependence of the maximum intensity and length of the pulse produced due to the decay of the packet at the leading edge of the envelope on the length of the optical fiber, and also the dynamics of the pulse spectrum. A typical decay of a multisoliton pulse is shown in Fig. 3.

Note that the pulse has no "pedestal" and that its steepening is considerable in comparison with the initial packet. These effects can therefore be used to drastically compress the pulses in the region of negative dispersion of the group velocity in optical fibers.

Pulses with $\tau_v \sim 200$ fs have recently been measured in this spectral region ($\lambda \sim 1.5\ \mu$m).[4,5] Our calculations have shown that a pulse with $\tau_v \sim 40$ fs ($\tau_v \sim 8 \cdot T$), in which approximately 60% of the total energy of the packet is concentrated, can be produced in an optical fiber of length $L \geqslant 100$ cm, even when the initial power of such a pulse is $P_0 = 6$ kW.

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